

Vibration-assisted fabrication of thin shells with spatially distributed imperfections

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Thin-shell structures, found in biological systems such as beetle carapaces and widely used in aerospace and civil engineering, achieve remarkable strength-to-mass ratios given their slenderness and curved geometries. However, their load-bearing capacity is highly sensitive to geometric imperfections, which are often unavoidable during fabrication and can trigger subcritical buckling. Silicone-based hemispherical domes have served as an experimental surrogate to study this phenomenon, yet prior work has largely focused on localized imperfections, failing to capture the spatially distributed nature of real-world imperfection patterns. Here, we introduce a vibration-assisted method for fabricating thin shells with spatially distributed, mode-shaped imperfections. Silicone is cast onto a thick elastic mold excited by a speaker, and vibration-induced flow during curing creates thickness variations. High-speed imaging and destructive measurements reveal material accumulation at the antinodes of the mold's vibrational modes. The engineered imperfections can be tuned by excitation frequency and mold shape, while their amplitude increases with speaker volume. Buckling experiments demonstrate significant reductions in critical pressure, offering a scalable platform to study and tune imperfection-sensitivity. Beyond shell mechanics, this method enables patterning of soft materials for applications ranging from morphable surfaces to bioinspired design.

Thin-shell structures are ubiquitous in both biological and engineered systems, with examples spanning a wide range of length scales, including virus capsids, pollen grains, beetle exoskeletons, pressure vessels, spacecrafts, and civil infrastructure^{1–4}. The mechanical advantage of thin shells stems from their curved geometry and large radius-to-thickness ratios. However, this slenderness also makes some shells highly susceptible to *subcritical buckling*, a sudden and catastrophic instability often triggered by small geometric imperfections.

This imperfection sensitivity has posed a fundamental challenge in shell mechanics for over a century. Early analyses by Zoelly⁵ provided the first closed-form expression for the critical pressure of an ideal, perfectly spherical shell, while subsequent developments by

Donnell and Batdorf established governing equations and stability criteria for cylindrical and spherical shells under external pressure^{6,7}. Von Kármán later demonstrated that equilibrium states involving large deflections could be sustained at loads far below the classical prediction^{8–10}. Tsien further attributed the discrepancy between theory and experiment to the extreme instability of the postbuckling path in the presence of arbitrary disturbances¹¹. A unifying theoretical framework was established by Koiter, whose asymptotic postbuckling theory demonstrated that infinitesimal geometric imperfections can induce dramatic reductions in critical load¹². Following this, extensive analytical and numerical studies sought to reconcile theory and experiment by examining the influence of thickness variations, non-uniform loading, boundary conditions, pre-buckling deformations,

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and geometric imperfections^{13,14}. Focusing on cylindrical shells, Babcock performed systematic comparisons of different imperfections and concluded that geometric imperfections dominate buckling sensitivity¹⁵.

Despite these advances, experimental progress has historically lagged behind theoretical and computational developments. As a result, practical shell design continues to rely on classical predictions augmented by empirical knockdown factors⁸. While careful experimental validation has been achieved for cylindrical shells and, in fewer cases, spherical shells^{12,13}, traditional fabrication methods often introduce uncontrolled and stochastic defect fields, making deterministic links between imperfection geometry and load-bearing capacity difficult to establish. This limitation has motivated statistical approaches to shell buckling¹⁶, as well as recent efforts to fabricate shells with controlled, prescribed imperfections^{17–20}.

The recent resurgence of interest in shell buckling is partly driven by the advent of novel fabrication and experimental techniques that enable the controlled introduction of imperfections into otherwise near-perfect shells²¹. In these works, the term “imperfections” is used in the shell mechanics sense to denote deviations from an idealized perfect geometry, including deliberately prescribed features introduced to probe imperfection-sensitive buckling behavior. Rather than experimenting with full-scale structures, desktop-sized silicone casts have been adopted as an experimental model system^{17,22}. These studies have established an experimental framework for imperfection-sensitive buckling of spherical shells, including quantitative prediction of knockdown factors for precisely engineered dimple-like defects²², characterization of large-amplitude defect plateaus²³, buckling under probing forces¹⁸, comparison of dimpled versus bumpy defects¹⁹, probabilistic buckling of shells with distributions of defects²⁴, and defect–defect interactions in multi-defect shells^{25,26}. While recent works have begun investigating multiple interacting imperfections^{25,27}, it remains difficult for such fabrication methods to reproduce the complex, global imperfection fields that arise naturally from manufacturing variability. Moreover, constructing dedicated fabrication setups for all possible imperfection geometries remains impractical.

Here, we introduce a vibration-assisted fabrication method to create thin hemispherical shells with prescribed global imperfection fields shaped by the vibrational modes of their casting mold. Vibrational excitations have long been used to impose patterns in physical systems. Chladni plate experiments, where fine particles accumulate

along the nodal lines of a vibrating surface, illustrate how modal patterns emerge from standing wave excitation²⁸. In fluid systems, Faraday waves are standing waves on a fluid surface excited by vertical vibration that create rich patterns through nonlinear resonances and symmetry breaking²⁹. In manufacturing, vibration has been harnessed to assist in powder spreading, colloidal self-assembly, and material redistribution in soft or particulate media^{30–34}. Central to these effects is the coupling between oscillatory driving and the medium’s response, often mediated by steady secondary streaming flows that persist over longer timescales than the driving frequency³⁵.

In our method, by actively exciting a thick elastic hemispherical mold with a speaker, and adjusting the excitation frequency and output volume, we induce resonant modal deformation of the mold’s surface during the curing of a thin liquid silicone layer. This vibration generates spatially structured flows within the liquid silicone, causing material to accumulate at antinodes and to thin elsewhere, thereby imprinting the mold’s vibrational displacement field as a permanent variation in shell thickness.

We characterize the resulting imperfection fields using stereographic optical imaging, reconstructing the full scalar field of shell thickness as a function of spherical coordinates. We find that these fields qualitatively resemble the vibrational eigenmodes as predicted by FE simulations, with modal symmetry and spatial distribution closely preserved. While the geometries of the imperfections are mode-dependent, the amplitude of thickness variation can be tuned continuously by varying the acoustic excitation volume. We then perform mechanical buckling experiments under vacuum loading to measure the effect of these globally distributed imperfections on structural stability. Lastly, we demonstrate that by altering the shape of the mold itself, a broader spectrum of imperfection shapes can be achieved.

Results

Vibration-assisted casting

To fabricate thin hemispherical shells with spatially distributed imperfections, we begin by casting a hemispherical elastic mold through repeated coating of a precisely machined metal sphere ($D = 50.8$ mm) with silicone (Mold Star 16 Fast Platinum Silicone Rubber) (Fig. 1a)¹⁷. By allowing the silicone to cure in-between successive coatings, the mold thickness t_m increases per coat by approximately 0.35 mm. Once we achieve a mold thickness that exceeds the thickness of the eventual hemispherical shells by an order of magnitude, we

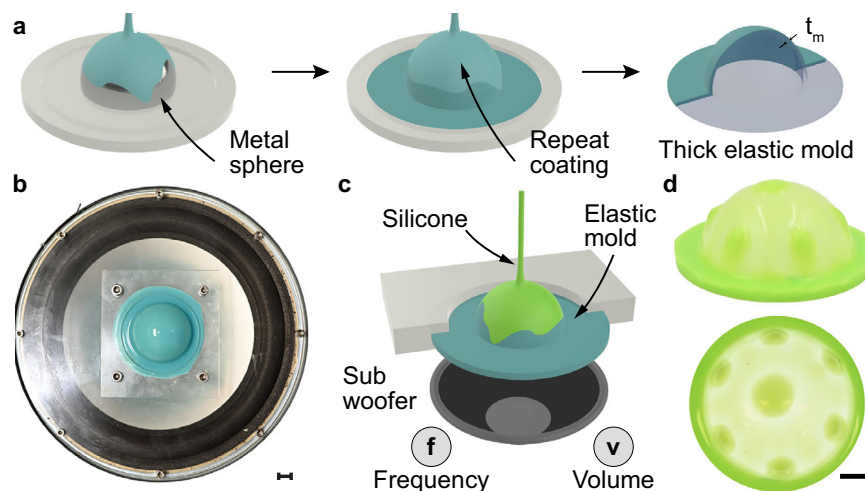


Fig. 1 | Overview of the casting and vibration-assisted fabrication protocol.

- a** Fabrication of an elastic mold by repeated coating of a metal sphere.
- b** Photograph of the mold attached to a PMMA plate bolted to the speaker.
- c** Casting of a silicone hemispherical shell atop the elastic mold as it is being excited

acoustically at a known frequency, f , and volume, v . **d** An example of a resulting imperfect shell with volume maximized to make visible the imperfections. The scale bars represent 10 mm.

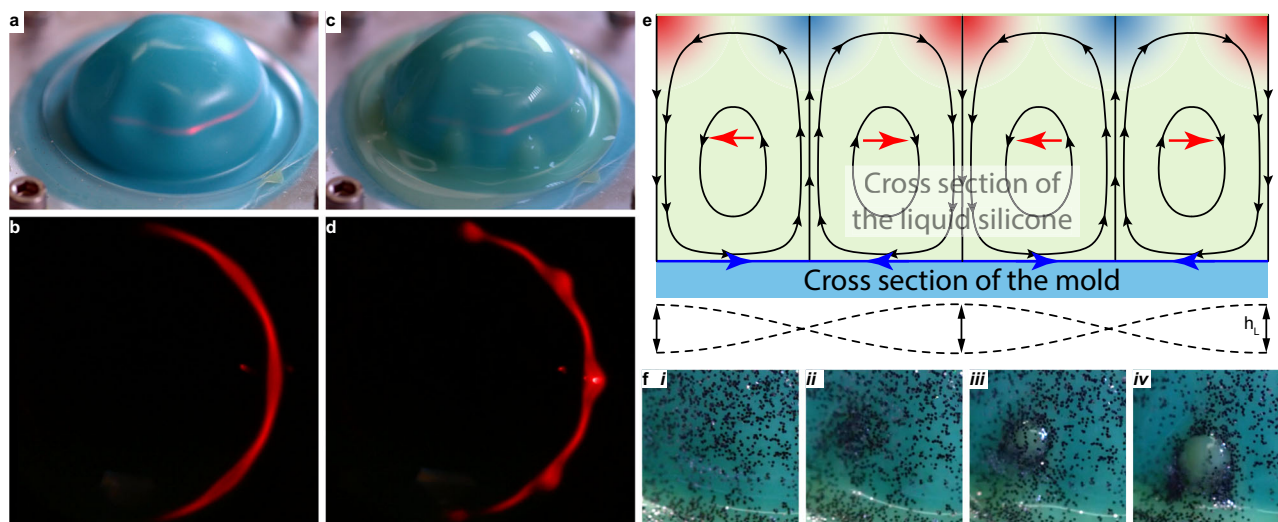


Fig. 2 | Physics of silicone casting under vibration. **a** Shell excited at a high speaker volume to make visible the periodic vibration as captured by a slow-motion camera (see SI Movie 1). **b** Projecting a horizontal laser sheet at a fixed elevation and photographing the vibrating mold from top down, we capture the locations of the nodes and antinodes of the mold. **c** Once silicone is poured onto the vibrating mold, multiple bumps are formed (see SI Movie 2). **d** With the same laser sheet, it is observed that the liquid silicone accumulates at the antinodes of the mold. **e** Sketch of the streaming flow's pressure distribution along one segment of one circumferential ring around the

shell spanning one wavelength of the vibration. Red indicates high pressure while blue indicates low pressure. The blue arrows indicate the boundary condition at the lower boundary, which leads to the circulating flow shown by the streamlines. The red arrows show the x-component of the forcing due to the Reynolds stresses. The vibration pattern is shown at the bottom for reference. **f** Stroboscopic slow motion capture (frame rate equal to the frequency of the speaker) shows a sequence of fluid flow tracked using reflective particles, i–iv captures progressive accumulation of liquid silicone at antinodes (see SI Movie 3).

proceed with casting. This thickness ratio ensures that the vibrational response of the mold is dominated by its own elastic properties and is minimally influenced by fluid-structure coupling with the yet-uncured silicone layer.

The hemispherical elastic mold is rigidly mounted onto a circular PMMA plate (≈ 230 mm in diameter) with a circular cutout in the center matching the mold's diameter. This plate is bolted to the rim of an acoustic speaker (Polk Audio PSW10 10" Powered Subwoofer) to form an enclosed volume (Fig. 1b). Prior to fabrication, we first determine the natural vibration modes of the elastic mold through Finite Element (FE) analysis. The frequencies used in the fabrication experiments are then chosen to coincide with these computed modal frequencies, allowing each shell to inherit the geometric signature of a specific vibrational mode. The frequency and volume of the speaker is digitally controlled with a PC. See SI Sec. 1 for the detailed fabrication protocol of the elastic mold.

To allow easy separation after casting, a mold-release (Ease Release 200, Smooth-On) is first sprayed on the elastic mold. Following this, a two-part vinyl polysiloxane (Elite Double 32, Zhermack) is mixed and poured onto the elastic mold to sufficiently cover the entire mold. The speaker is then powered on at a specific frequency and volume (Fig. 1c). The acoustics induce vibration in the mold, which redistributes the still-liquid silicone into a new steady-state prior to solidification (see SI Movie 1). As the silicone cures, this redistribution becomes permanently imprinted in the shell geometry. An example of such spatially distributed imperfections is shown in Fig. 1d. By tuning the frequency and the volume, we are able to fabricate shells of different imperfection geometries.

Note that the timescale of curing (~ 20 min) is much slower than the formation of the imperfection pattern (~ 1 to 5 s), which is itself much slower than the period of the excitation (≤ 0.01 s). Further, the exponential nature of curing suggests that, during the initial period following pouring, the silicone behaves predominantly as a viscous Newtonian fluid and does not exhibit shear-thickening or shear-thinning behavior^{17,36}.

The significant separation of timescales between vibration-driven redistribution (~ 1 to 5 s) and curing (~ 20 min) implies that the thickness

patterns form well before solidification. The final shell geometry is therefore established during the liquid phase, while curing primarily serves to “freeze” an already-formed thickness distribution. Accordingly, the observed pattern formation is governed by vibration-induced flow.

Imperfection pattern formation

We proceed to understand the formation of imperfections. Specifically, we use slow-motion videography to capture how the vibration of the mold shapes the liquid silicone as it is poured over (see SI Movie 2). We choose one modal frequency $f = 167$ Hz as informed by FE analysis featuring 8 nodes and 8 antinodes distributed around the circumference. By adjusting the speaker volume, we observe periodic deformations on the mold matching the modal shape from the FE (Fig. 2a). By setting the frequency of video capture to match that of the speaker frequency, the resulting stroboscopic video shows negligible movement, demonstrating that the mold is vibrating in sync with the speaker (see SI Movie 2).

To experimentally identify nodal and antinodal locations, we illuminate the vibrating mold with a laser sheet positioned at a fixed elevation $z = 0.25r$. The laser sheet intersects the mold surface along a curve; we record the motion of this intersection curve around the circumference using time-integrated imaging. Locations with minimal motion are identified as nodes, whereas locations with maximal motion are identified as antinodes (Fig. 2b).

Once silicone is poured onto the mold, the onset of acoustic-driven vibration causes the uncured silicone to flow and accumulate at distinct points (Fig. 2c). The same laser sheet photography from the top view shows that the accumulations coincide with the locations of the antinodes (Fig. 2d), i.e., where the magnitude of the vibration amplitude is at a maximum in a standing-wave pattern.

This seems counterintuitive as the antinodes experience the largest vibration amplitude and could cause the liquid to flow away. A theoretical analysis of the flow reveals that the mold's vibrations drive an oscillatory primary flow of the same frequency and wavelength as the vibrations. The primary flow creates Reynolds stresses with half the wavelength that, together with a boundary forcing created by the

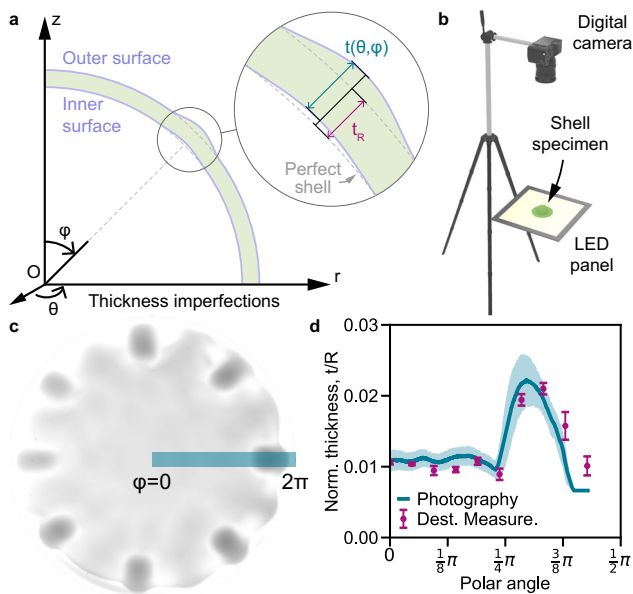


Fig. 3 | Quantification of vibration-induced imperfection as a measure of shell thickness. **a** Schematic of thickness measurement per spherical coordinate. **b** Photographic stage used to infer the shell's thickness field. **c** Processed photograph of an imperfect shell featuring eight bump-like defects. **d** Comparison between inferred thickness from photography and destructive measurements of the same shell. Source data are provided as a Source data file.

primary flow, drive a secondary, time-averaged, streaming flow with half the wavelength (Fig. 2e). The associated pressure would push an undeformed free surface outward at the locations of the antinodes, leading the liquid silicone to redistribute toward them. See SI Sec. 5 for more details.

To experimentally visualize the dynamics of this flow, we introduce reflective particles on the surface of the uncured silicone. Slow-motion imaging captures the progressive migration of particles toward antinodes during curing from all directions on the surface (Fig. 2f), driven by a secondary mean streaming flow, as predicted by the theory. Once the bumps are formed over a time period ~ 5 s (that is much longer than the period of the vibration), an internal toroidal circulation is observed from the motion of the particles that matches those predicted in Fig. 2e. Similar to vortex rings, the liquid silicone moves to the circumference of the bump before moving under and emerging at the apex (see SI Movie 3). This secondary streaming flow is much weaker than the primary oscillatory flow, and bears resemblance to the streaming flow associated with Faraday waves³⁵. Here, gravity may bias the overall symmetry of the flow by acting as a body force on the accumulating fluid. If the speaker is shut off or tuned to another frequency, the fluid migrates accordingly until curing occurs in approximately 20 min, at which point the shell is peeled off the mold. This allows us to correlate the modal vibration to the eventual shape of the shells cast upon the elastic mold.

Quantification of imperfection field

To quantify the geometry of the imperfect shells fabricated using the above method, we reconstruct their thickness profile using optical photography. We observe that the vibration-generated imperfection generally takes the form of thickness variations across the surface of the shell (Fig. 3a). Further, deviations predominately occur on the outer surface of the shell; the inner surface remains spherical. We quantify the thickness of an imperfect shell at every spherical coordinate as $t(\theta, \phi) = d_{\text{outer}}(\theta, \phi) - d_{\text{inner}}(\theta, \phi)$ where d_{outer} and d_{inner} are the distances from the outer and inner surfaces of the shell to the origin, respectively.

We simplify the process using optical photography as a measuring tool. We position each fabricated shell atop an LED light panel with constant ambient lighting. A digital camera (Nikon D780, 105 mm f/2.8G) is placed vertically above the shell at distance of 400 mm (Fig. 3b). Each resulting image is processed to remove optical distortion and then stereographically mapped to 2D (Fig. 3c)³⁷. The degree of light penetration through the silicone serves as an indirect measure of thickness. With the exposure set to ensure no highlight or shadow clipping, i.e., no pure white or black, we are able to fit the relationship between intensity and thickness with an attenuation coefficient³⁸.

To quantify the entire thickness field (see SI Sec. 2.2 for the image processing algorithm), we provide independent measures of the maximum and minimum thicknesses ($t_{\text{max}}, t_{\text{min}}$). This is achieved by excising segments of the fabricated shell and directly measuring their thickness using an optical microscope (Fig. 3d, see SI Sec. 2.1 for the destructive measurement method). These findings confirm that the vibration introduces repeatable, spatially structured imperfections whose geometry closely resembles vibrational mode shapes. See SI Fig. 2 for a library of different imperfection patterns.

Note that even in the absence of vibration, static casting of thin shells can produce axisymmetric thickness variations along the polar angle due to gravitational drainage and surface tension effects. For the silicone used in this study (Elite Double 32), Lee et al.¹⁷ reported baseline thickness deviations of approximately $\pm 6.6\%$ relative to the mean thickness. Previous studies have shown that such uncontrolled, axisymmetric thickness variations have a weak influence on the critical buckling pressure compared to intentionally introduced geometric imperfections²². More recent work further demonstrates that when multiple imperfections are present and sufficiently separated, the largest imperfection dominates the buckling response²⁵.

To ensure that this baseline effect does not influence our conclusions, we fabricated additional near-perfect shells without acoustic excitation using the identical experimental configuration. Measurements confirm that the polar thickness variation in these static shells remains within the same range reported in the literature and does not introduce significant non-axisymmetric features (see SI Fig. 8).

To assess repeatability, we fabricated multiple shells under identical excitation conditions and quantified the similarity of the resulting imperfection fields. Three near-perfect shells and three shells with eight bump-like defects were fabricated using multiple molds and photographed under identical conditions. Pairwise Pearson correlation coefficients were computed between the grayscale thickness maps of all specimens (see SI Fig. 3). Within each group, the mean correlation coefficients are 0.936 for near-perfect shells and 0.638 for shells with eight defects, indicating consistent reproduction of the targeted pattern. In contrast, correlations across the two groups are negligible (≈ -0.12).

Tuning imperfection geometry

We have shown earlier that the elastic mold vibrates in sync with the frequency of the speaker (Fig. 2a). This allows us to use FE simulations to predict and engineer the vibrational modes of the elastic mold as a function of the speaker frequency. Specifically, we calculate the natural vibrational modes and frequencies of the elastic mold with the equatorial degrees of freedom constrained. We select three different modal shapes for experimentation, namely $f = 153, 167$, and 210 Hz. These exhibit a single ring of six, eight, and ten antinodes, respectively. For each mode, we normalize the displacement field by its maximum absolute radial displacement, so that the largest deviation from the undeformed configuration equals one. We project the resulting displacement fields to 2D for visual comparison (Fig. 4a). See SI Sec. 3 for the detailed FE analysis protocol.

We then set the speaker to the corresponding frequencies and cast the shells using the aforementioned method. Although the FE

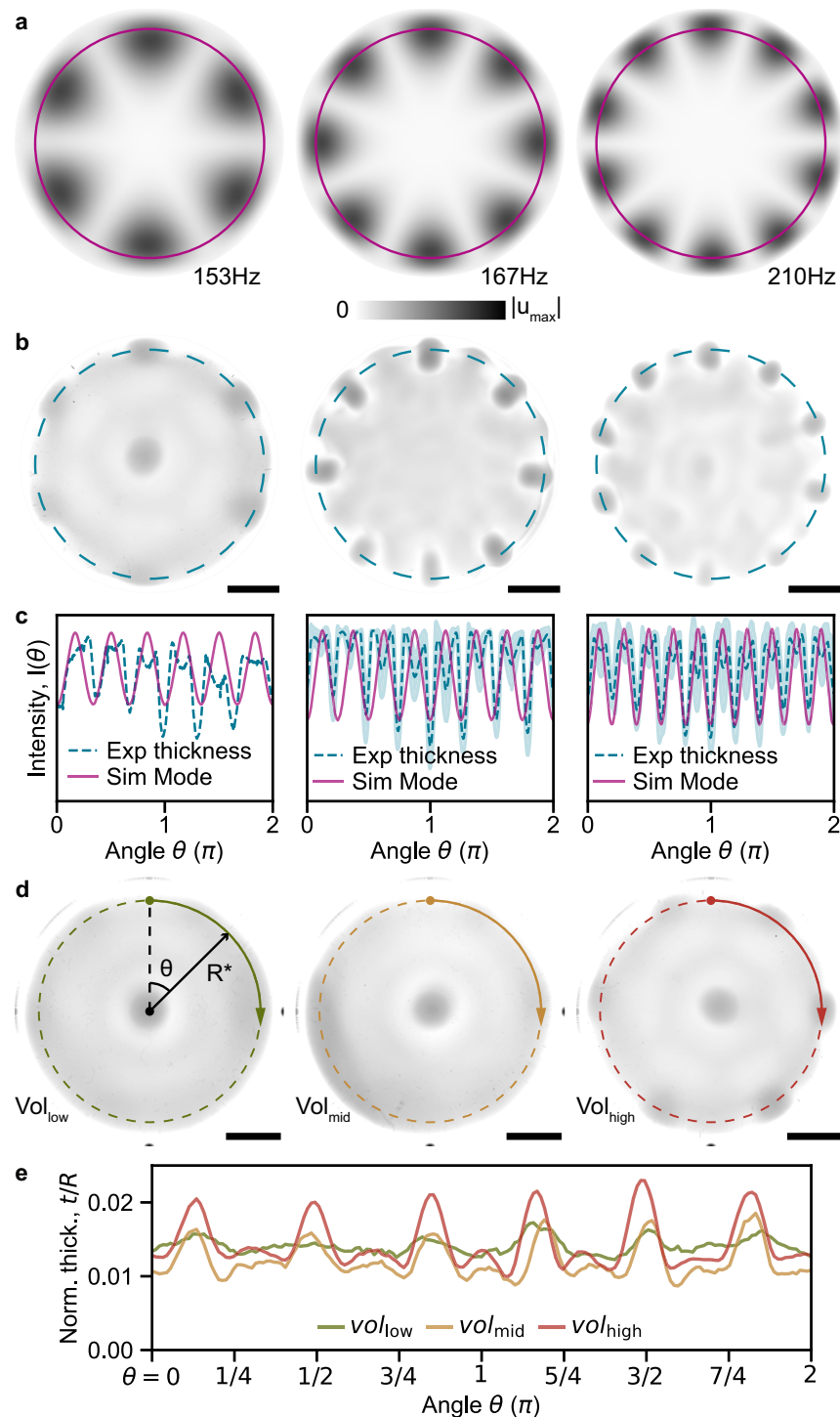


Fig. 4 | Comparisons between FE-predicted modes and the experimental imperfection patterns. a Simulated modal shapes for three distinct vibrational modes, showing radial deviation fields mapped onto 2D. **b** Photographs of the fabricated shells showing global shape deviations and mode symmetry. **c** Inferred intensity of the photographs as compared to the modal shape displacements,

showing the correct number and distribution of peaks. **d** By varying the speaker volume and setting a constant frequency, we tune the amplitude of the thickness field $t(\theta, \phi)$. **e** The thickness distribution at a single elevation is plotted for shells fabricated at three different volumes. All scale bars are 1 cm. Source data are provided as a Source data file.

vibrational mode shapes represent displacement fields of the mold and the experimentally measured shell geometries represent thickness variations, both are shown to share the same underlying spatial symmetry and antinode distribution (Fig. 4b). These observations support that vibrations can reliably embed modal features as geometric imperfections in thin shells. To quantitatively assess this correspondence, we compare the circumferential intensity patterns of the FE-

predicted modes and the fabricated shells along a circular ring near the perimeter at $r = 0.85R$. For each specimen, the intensity is averaged along this ring and plotted as a function of azimuthal angle θ . The resulting profiles exhibit matching periodicity between simulation and experiment, with root-mean-square errors of 0.0798, 0.127, and 0.0924 for the 6-, 8-, and 10-defect cases, respectively. See Fig. 4c and SI Sec. 3.1 for details.

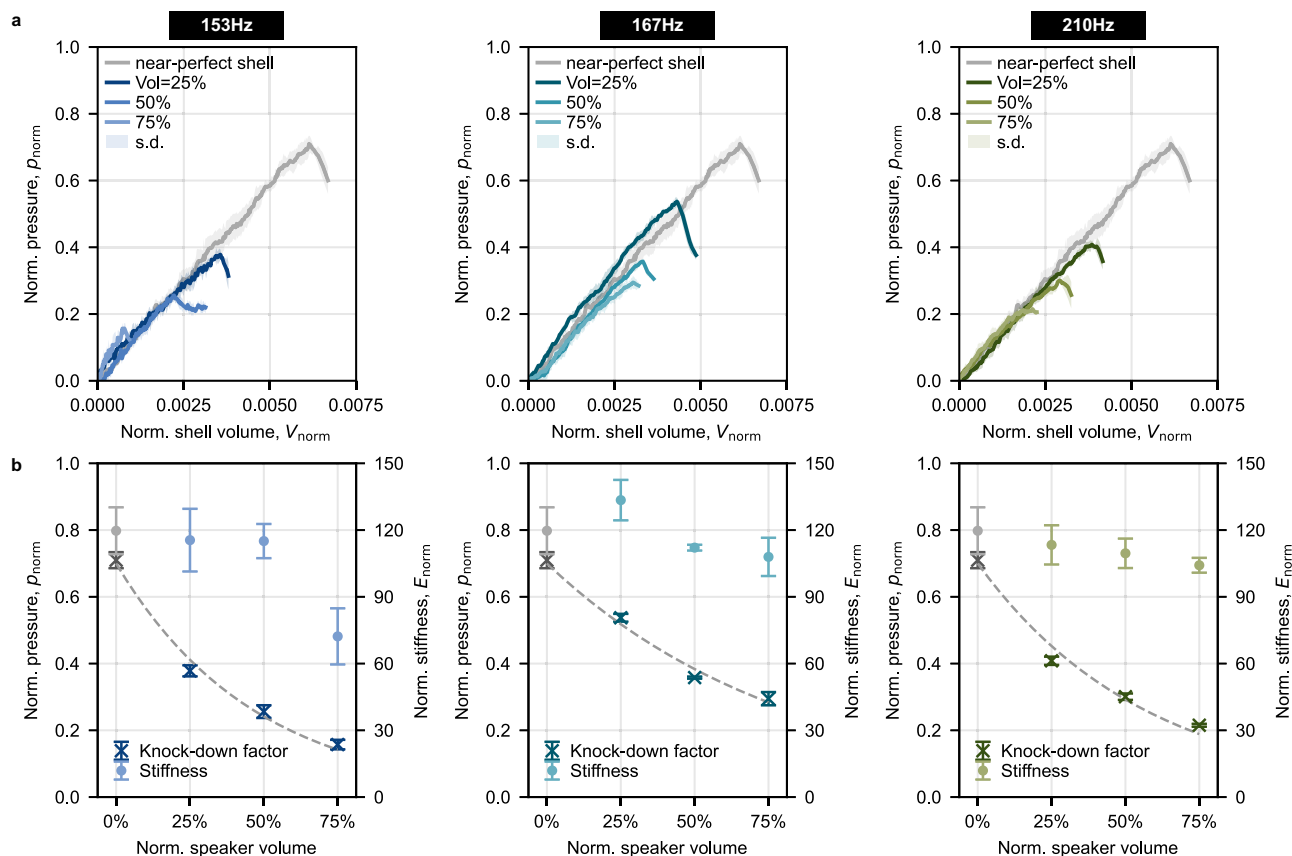


Fig. 5 | The buckling response of hemispherical shells with vibration-induced imperfections. a Normalized pressure–volume curves for shells fabricated with specific speaker excitation volume and frequencies. The behavior of a shell with no vibration-induced imperfections is plotted for reference. **b** Normalized pre-

buckling stiffness E_{norm} and knockdown factors as a function of speaker volume, quantifying the reduction in structural stability with increasing imperfection severity. Source data are provided as a Source data file.

To further quantify periodicity, we compute the discrete Fourier transform of the azimuthal intensity profiles. The resulting power spectra show dominant peaks at harmonic indices corresponding to the number of antinodes in the targeted vibrational mode, both in the FE predictions and in the fabricated shells (SI Fig. 10d). Additional lower-amplitude harmonics reflect global asymmetry and nonlinearities at higher excitation amplitudes. A fully predictive description that couples acoustic excitation, structural vibration, oscillation-driven flow, and curing kinetics will be conducted in the future to quantitatively predict this correlation.

Next, we modulate the speaker volume, $v = [55\%, 69\%, 72\%]$, while keeping the frequency constant at $f = 153$ Hz. The resulting shells exhibit the same imperfection patterns qualitatively, however, the amplitude of thickness variation increases systematically (Fig. 4d). Plotting the thickness distribution along a single latitude shows six peaks in all three fabricated shells. The thickness profiles exhibit smooth, spatially graded variations that decay away from the antinodal peaks. This graded character is consistent with the spatial envelope of the FE-predicted modal displacement field. Their amplitude increases systematically as a function of the speaker volume.

By modulating both frequency and volume, we are able to systematically change imperfection geometry and severity. In principle, we have access to all modes as predicted by FE; however, experimentally, physically achievable eigenmodes are governed by several coupled experimental factors. First, the subwoofer used in this study accurately reproduces low-frequency excitation (40–250 Hz), while higher-order modes would require a mid-range or tweeter-type driver. Second, because the silicone mold is viscoelastic, maintaining the same vibration amplitude at higher frequencies would incur greater strain energy, which

requires greater input power. If the speaker output remains constant, the amplitude decreases with frequency until the vibration becomes insufficient to influence the liquid layer. Finally, the secondary streaming flow that drives material redistribution penetrates a depth proportional to the vibrational wavelength. As the distance between neighboring antinodes decreases for higher modes, this flow becomes confined near the surface of the silicone mold and its magnitude diminishes.

Mechanical response of imperfect shells

We quantify the mechanical response of this class of imperfect shells under increasing external pressure, achieved by progressively reducing the internal pressure of the shell (Fig. 5). In addition to perfect shells, a total of nine different imperfect hemispherical shells are fabricated and divided into three groups, each subjected to acoustic excitation at a fixed frequency listed in Fig. 4a. Within each group, the shells are exposed to varying excitation amplitudes by adjusting the speaker's output volume. Quasi-static under-pressure experiments are performed on the shells. The interior of each shell is connected to an automated syringe pump via a flexible tube. The volume in the shell V_{shell} , the tube V_{tube} and the 1 mL syringe V_{syr} form a closed system with a total volume V_{total} . A pressure sensor is used to record pressure as the syringe draws air and expands the total volume (see SI Sec. 4.1 for detailed experimental setup). As the total volume increases, the pressure decreases until the shell buckles, at which point the volume decreases abruptly and the pressure rebounds.

Assuming the ideal gas law holds, we can isolate the volume under the shell (see SI Sec. 4.2 for calculation). This allows us to define a normalized volume V_{norm} as $\Delta V/V_{\text{shell}}$, where $\Delta V = V_{\text{shell}} - V_{\text{shell}}^{\text{init}}$. We further define a normalized internal pressure as $p_{\text{norm}} = p/p_{\text{crit}}$, where

p_{crit} is the under-pressure required to buckle a perfect hemispherical shell. We proceed to plot the normalized internal pressure as a function of normalized volume change (Fig. 5a). Each PV curve corresponds to a specific excitation volume and frequency. Increasing the speaker volume results in a systematic reduction in critical buckling pressure and earlier onset of instability.

We further quantify the stiffness as the slope of the quasi-linear portion of the normalized pressure–volume curves prior to the onset of instability (Fig. 5b). We compute a normalized stiffness as $E_{\text{norm}} = p_{\text{norm}}/V_{\text{norm}}$ over the initial linear regime ($V_{\text{norm}} = 0.002$). Across all excitation frequencies and volumes, the measured stiffness values remain within the experimental variability of the near-perfect shell, indicating that the imposed imperfections primarily affect the critical buckling pressure rather than the pre-buckling stiffness. A deviation is observed only for shells with six defects fabricated at the highest excitation volume, where large thickness variations likely violate the assumptions of thin-shell behavior. See SI Table 1 for numerical values.

We then calculate the corresponding knockdown factors κ of each shell as the ratio between the maximum buckling pressure of imperfect shells and that of the theoretical prediction of a perfect shell (see SI Sec. 4.3 for characterization of a perfect shell). These are plotted against speaker volume to confirm a consistent decline in structural capacity with stronger perturbation volume (Fig. 5b). To further study the relationship between excitation amplitude and structural capacity, we fit exponential decay curves of the form $\kappa = \alpha \exp(-k \cdot \nu)$ to the measured knockdown factors, where ν is the normalized speaker volume. Across all three modal families, the data are well described by a shared pre-factor $\alpha = 0.7$, with mode-dependent decay constants $k_6 = 0.532$, $k_8 = 0.301$, and $k_{10} = 0.437$ ($R^2 = 0.991, 0.988, 0.980$, respectively). The shared pre-factor suggests that the baseline knockdown factor at low excitation is mode-independent, while the differing decay constants reflect the mode-dependent sensitivity of the buckling response to increasing imperfection severity. These fits are shown in Fig. 5b.

Discussion

We have introduced a vibration-assisted fabrication method to create hemispherical shells with spatially distributed, mode-shaped geometric imperfections in the form of thickness variations. By casting liquid silicone atop a vibrating elastic mold, we exploit vibration-driven flow and secondary streaming to imprint modal shapes directly onto the shell's thickness field. The resulting imperfections are tunable in geometry via excitation frequency and scalable in severity through speaker volume, enabling controlled access to classes of imperfection patterns that are difficult to realize using conventional fabrication approaches.

To explore the underlying mechanism, we address the following physical processes. Analysis of the vibration-driven fluid motion reveals a secondary streaming flow that drives material accumulation at regions of large vibration amplitude. Quantitative correlation analysis between the measured imperfection fields and FE-predicted vibrational modes demonstrates that the vibration-assisted patterning achieves high spatial fidelity and repeatability across shells fabricated under identical conditions. Mechanical buckling experiments demonstrate a systematic reduction in structural capacity with increasing imperfection amplitude.

Beyond single-mode excitation, this fabrication method enables the generation of mixed-mode and non-periodic imperfection fields through broadband excitation. By exciting the mold with broadband signals, the resulting thickness variations reflect both superposed and emerging modal features. The resulting shell geometries exhibit features both resembling the single-mode patterns shown in Fig. 4 and previously unobserved spatial patterns. Representative examples and experimental details are provided in the SI Sec. 1.6. The ability to imprint complex thickness landscapes highlights the flexibility of vibration-assisted patterning and suggests routes toward exploring the mechanics of imperfection interactions in thin shells.

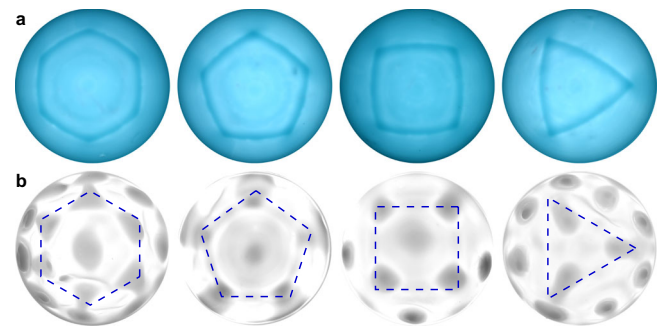


Fig. 6 | Imperfection shapes produced by altering the shape of the elastic hemispherical mold. a Protrusions of different shapes, including a hexagon, pentagon, square, and triangle, are embedded onto the mold. **b** The resulting imperfection shapes show silicone accumulating at the vertices of the mold polygons.

Furthermore, by pre-designing the mold geometry with polygonal features (i.e., triangular, square, pentagonal and hexagonal protrusions), the vibrational response of the mold can be designed to produce non-standard imperfection patterns, including configurations with odd numbers of bump-like defects (see Fig. 6 and SI Sec. 1.7). This demonstrates that combining mold geometry design with structural vibration significantly broadens the accessible design space of imperfection geometries.

While the present work focuses on hemispherical shells as a canonical system for imperfection-sensitive buckling, the underlying fabrication method is readily extendable to programming the mechanics of other soft material systems^{39,40}. Here, we reinterpret thickness imperfections as architected features, and leverage the spatially distributed nature of such features as a design variable. Such features may be used in localizing compliance, biasing snap-through behavior, or encoding preferred post-buckling modes^{41,42}.

In programmable materials and soft robotics, geometric instabilities are increasingly exploited to achieve large, rapid shape changes, mechanical logic, or passive actuation without embedded electronics^{43–45}. Examples include morphable shells that snap between configurations, soft robotic grippers that close via elastic instability, and surfaces that reversibly change curvature under pressure^{46–48}. The ability to imprint global, mode-shaped thickness fields provides a means to pre-program these behaviors directly into the material geometry, rather than relying on external constraints or active control. In particular, recent studies have shown that spatial disorder itself can be treated as a programmable design variable, enabling tunable mechanical response and emergent behavior in elastic systems^{49–51}.

In conclusion, this study establishes vibration-assisted fabrication as an enabling platform for experimentally accessing distributed, tunable geometric features that reflect the complex defect landscapes encountered in real-world structures.

Methods

Fabrication

Elastic hemispherical molds are fabricated by repeatedly coating a precision-machined metal sphere ($D = 50.8$ mm) with a platinum-catalyzed silicone rubber (Mold Star 16 Fast, Smooth-On; PSR16), adding ≈ 0.35 mm per cure cycle. The Young's modulus of PSR16 ($E = 0.85$ MPa) is measured by tensile testing of ASTM D412 dog-bones. To cast imperfect shells, the mold is mounted on a PMMA plate bolted to the rim of an acoustic subwoofer (Polk Audio PSW10), sprayed with a mold-release (Ease Release 200), and coated with a two-part vinyl polysiloxane (Elite Double 32, Zhermack; VPS32) while the speaker is driven at a prescribed frequency and volume. Mold vibration redistributes the liquid silicone into a steady thickness pattern that is fixed in place over the ~ 20 min cure cycle.

Imperfection geometry is set by the excitation frequency (chosen to coincide with a computed modal frequency of the mold; see below) and amplitude by the speaker volume. Mixed-mode patterns are generated using broadband audio signals, and non-standard patterns are produced by casting the mold against 3D-printed hemispherical negatives containing polygonal indentations. Full protocols, including material characterization and flow visualization (laser-sheet node/antinode identification, stroboscopic imaging, and reflective-particle tracking), are given in SI Sec. 1.

Thickness measurement

Ground-truth thickness profiles are obtained destructively by excising strip segments and measuring them under an optical microscope. For non-destructive reconstruction, each shell is photographed from vertically above atop an LED light panel using a DSLR camera (Nikon D780, Nikkor 105 mm f/2.8G) under controlled lighting. Grayscale images are corrected for geometric edge-darkening and inverse-orthographically remapped, and local thickness is inferred via the Beer-Lambert law with the attenuation coefficient and incident intensity fit against the destructive measurements. Repeatability is assessed by Pearson correlation of grayscale thickness maps across independently fabricated specimens. Full calibration details are provided in SI Sec. 2.

Finite element modal analysis

Modal analysis of the elastic mold is performed in Abaqus/Standard using reduced-integration shell elements (S4R) on a perfect hemisphere ($R = 25.4$ mm) with the equator fully constrained and the measured PSR16 properties ($E = 0.85$ MPa, $\nu = 0.5$). The first 30 eigenmodes are extracted using the Lanczos solver. Modes with 6, 8, and 10 circumferential antinodes ($f = 153, 167$, and 210 Hz) are selected for fabrication. FE-predicted modes and fabricated shells are compared via root-mean-square error and discrete Fourier decomposition of circumferential intensity profiles at $r = 0.85R$. See SI Sec. 3.

Mechanical buckling experiments

Quasi-static under-pressure buckling tests are performed in a closed-volume pneumatic setup comprising the air beneath the shell, flexible tubing, and an automated 1 mL syringe pump, sealed between two clamped PMMA plates. Internal pressure is monitored with an MPL3115A2 barometric sensor (ELEGOO UNO R3 microcontroller, 0.8 s sampling). The syringe slowly withdraws air until the shell buckles; the volume beneath the shell at each timepoint is recovered from the measured pressure via the ideal gas law. Pressure is normalized by the classical critical pressure $p_{\text{crit}} = 2E/\sqrt{3(1-\nu^2)} \cdot (t/R)^2 = 102.4$ Pa for our VPS32 shells ($E = 1.2$ MPa, $\nu = 0.5$, $t = 0.2$ mm, $R = 25.4$ mm), and volume by the initial shell volume. Pre-buckling stiffness is computed as the slope of the normalized pressure–volume curve over the initial linear regime. Knockdown factors are fit to $\kappa = \alpha \exp(-\kappa\nu)$ by nonlinear least-squares regression. Three shells are tested per excitation condition. See SI Sec. 4.

Weakly nonlinear analysis of the vibration-induced flow

To explain why liquid silicone accumulates at the mold's antinodes with a free-surface wavenumber twice that of the mold's vibration, we perform a weakly nonlinear analysis of a Newtonian fluid layer driven by a prescribed standing-wave deflection of the lower boundary. Expanding the velocity, pressure, and free-surface fields in powers of the vibration amplitude, the order- ϵ' primary flow is linear and oscillatory, while the order- ϵ'^2 secondary flow is driven by Reynolds stresses and a boundary forcing with wavenumber $2k'$. Symmetry arguments fix the spatial phase, yielding a streaming flow with pressure maxima above the antinodes that deflects the free surface outward there. In the experimental parameter regime ($Re = \mathcal{O}(10^{-2})$, $Ca = \mathcal{O}(10^2)$,

and $k' = \mathcal{O}(10^{-1})$), the steady free-surface amplitude scales as $A - \lambda^2 \mu^2 \omega^2 \epsilon'^2 \gamma^{-2} H^{-1}$. The full derivation is provided in SI Sec. 5.

Data availability

The source data underlying all figures presented in this study are provided with this paper as a Source data file. The complete dataset has been deposited in the Zenodo public repository and is available at <https://doi.org/10.5281/zenodo.19682356>⁵². No datasets with restricted access are associated with this study. Source data are provided with this paper.

Code availability

All custom scripts for figure generation are deposited in the same Zenodo repository (<https://doi.org/10.5281/zenodo.19682356>)⁵². Scripts are developed and tested with Python 3.10 (NumPy, pandas, SciPy, Matplotlib, OpenCV).

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Author contributions

T.C. designed and directed the research. I.K. performed the experimental and numerical research with J.T.’s assistance. L.M. and D.F. provided the analysis on the fluid-structure interaction. All authors drafted and edited the manuscript.

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Competing interests

The authors declare no competing interests.

Additional information

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Supplementary Information

Vibration-Assisted Fabrication of Thin Shells with Spatially Distributed Imperfections

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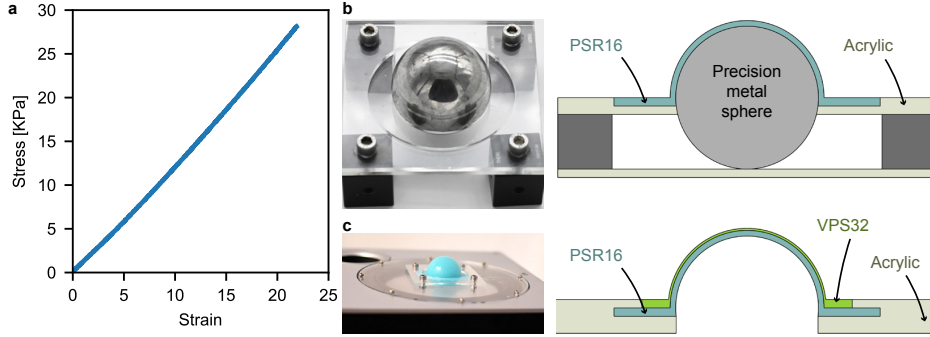
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1 Fabrication protocol

1.1 Material characterization

Two materials are used in this study. Mold Star 16 Fast Platinum Silicone Rubber (PSR16) for fabrication of the mold, and Zhermack Elite 32 (VPS32) for the fabrication of the imperfect shells. To predict the vibrational characteristics of the mold using Finite Element (FE), we conduct material characterization of the PSR16 material. We cast dogbones according to ASTM D412, type D geometry, and subject them to tension up to a strain of 0.02 at a displacement rate of 0.15 mm s^{-1} . We capture the initial portion of the stress-strain curve (S.Fig. 1a) and calculate the Young's modulus of PSR16 to be $E = 0.85 \text{ MPa}$.



Supplementary Figure 1: **Fabrication of the mold and of the imperfect shells.** a. Stress-strain plot of the tensile test of mold material PSR16. b. Physical setup of the mold fabrication. Cross section showing the cavity upon which PSR16 is poured. c. Physical setup of the acoustic-assisted fabrication of the imperfect shells. Cross sectional schematic showing where the VPS32 is poured. Source data are provided as a Source Data file.

1.2 Fabrication of elastic hemispherical molds

The elastic hemispherical mold used in the acoustic casting is fabricated by repeatedly coating a precision-machined metal sphere (diameter $D = 50.8 \text{ mm}$) with a platinum-catalyzed silicone rubber (Mold Star 16 Fast, Smooth-On). The setup positions the metal sphere using a PMMA sheet (S.Fig. 1). To prepare the mixture for pouring, we use a centrifugal mixer (ARE-250, Thinky Corporation) to homogenize the base and catalyst. The two-part mixtures of PSR16 are prepared at a 1:1 weight ratio of base to catalyst. Each mixture is mixed for 30 seconds at 2000 rpm in the clockwise direction, followed by another 30 seconds at 2000 rpm counterclockwise. Each layer is poured and allowed to fully cure for 20 minutes at room temperature before the next is added. Each curing cycle adds approximately 0.35 mm to the mold thickness.

1.3 Fabrication of imperfect hemispherical shells with acoustic excitation

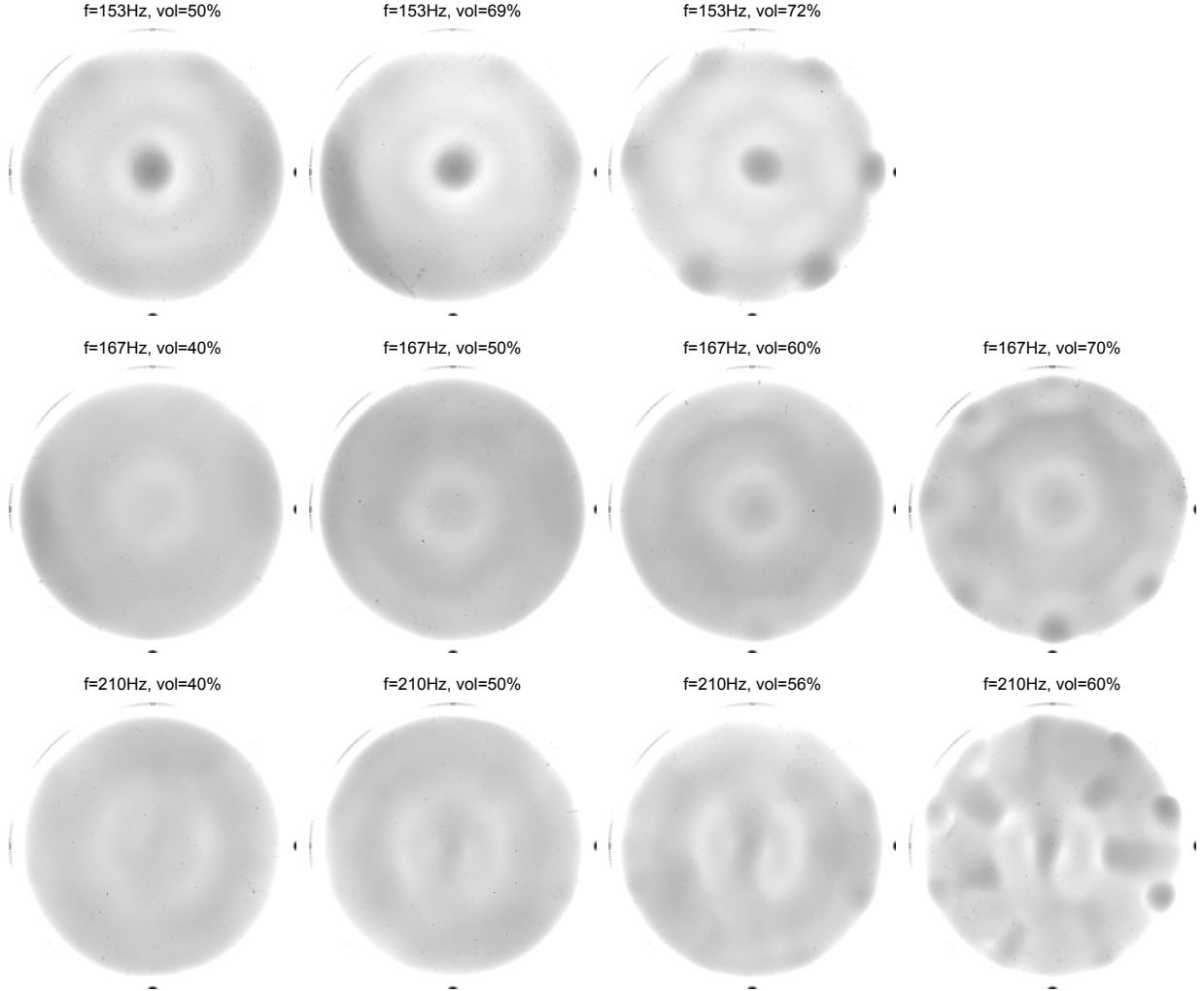
Once the mold is fabricated, it is affixed using a PMMA sheet to an acoustic subwoofer (Polk Audio PSW10 10" Powered Subwoofer). The PMMA sheet clamps around the PSR16 elastic mold and has a circular cut out matching the diameter of the mold underneath (S.Fig. 1b).

Prior to each casting, the mold is sprayed with an aerosol-based mold release (Mann Release Technologies, Ease Release 200). Silicone (Zhermack Elite 32) prepared with the same procedure as above is then poured over the PSR16 elastic mold while the speaker is turned on at a prescribed

frequency and volume. This continues until the silicone fully cures after 20 minutes, and the imperfect shell is peeled off the elastic mold.

1.4 Different imperfect shells with varying speaker frequency and volume

By changing the frequency of the acoustic speaker, we can excite different vibrational modes of the mold. Conversely, by modulating the speaker volume, we can change the amplitude of that vibration. S.Figure 2 shows photographically the effect of the different frequencies and volumes on the eventual shape of the imperfection.



Supplementary Figure 2: **Top and side views of the elastic hemispherical shell responding to acoustic excitation at different frequencies and speaker volumes.** Each column shows the shell's shape under a specific excitation condition, with the frequency and volume noted above. The top view of each specimen shows the vibrational mode patterns, while the bottom image shows the same shell from the side.

1.5 Robustness and repeatability

To demonstrate the robustness and repeatability of the fabrication method, we fabricate three near-perfect and three shells with 8 bump-like defects, and calculate the Pearson correlation coefficient within each group. Note that since the lighting condition of the photographs are identical, the intensity of the pixels of the gray-scale images are compared directly. Each pair of images are treated as vectors of pixel intensities. Each image is first converted to a one-dimensional vector by stacking all pixels, and its mean intensity is subtracted to remove global brightness offsets. The Pearson correlation coefficient p is then calculated as the normalized inner product of the mean-centered vectors, obtained by dividing their dot product by the product of their Euclidean norms (Eq. 1),

$$p(I_1, I_2) = \frac{\sum_{i=1}^N (I_{1,i} - \bar{I}_1)(I_{2,i} - \bar{I}_2)}{\sqrt{\sum_{i=1}^N (I_{1,i} - \bar{I}_1)^2} \sqrt{\sum_{i=1}^N (I_{2,i} - \bar{I}_2)^2}} \quad (1)$$

, where I_1 and I_2 are the vectors of pixel intensities of two images being correlated, and N is the number of pixels within the circular mask of the photographs. This normalization renders the metric invariant to global intensity scaling and offset.

The correlation between three near-perfect shells and three shells with 8 bump-like defects are calculated based on Eq. 1 (S.Fig. 3). Within each group, the correlation means are $p_0 = 0.936$ and $p_8 = 0.638$ for near-perfect and 8 defects respectively, with $p = 1$ representing a perfect correlation. The deviations within each group can be attributed to three primary causes. The first is the time lapsed between silicone mixing and pouring. This delay influences the eventual thickness of the specimen [1] and may explain the difference between t8-s3 and the remaining two. The second is the imprecision in positioning and leveling the mold on the speaker. The mold is secured onto a circular PMMA plate, which is bolted to the rim of the loudspeaker. In this setup, the mold may not be perfectly concentric to the diaphragm of the speaker. Depending on the tightness of the bolts, the lip of the mold may be asymmetrically compressed. Both errors may result in the specimen appearing asymmetric. The third is in image processing, specifically, we manually align one bump-like defect along the horizontal axis by rotating the image. Nevertheless, the within-group correlations are substantially higher than cross-group correlations ($p_{0-8} = -0.117$), confirming that the fabrication process produces consistent imperfection patterns.

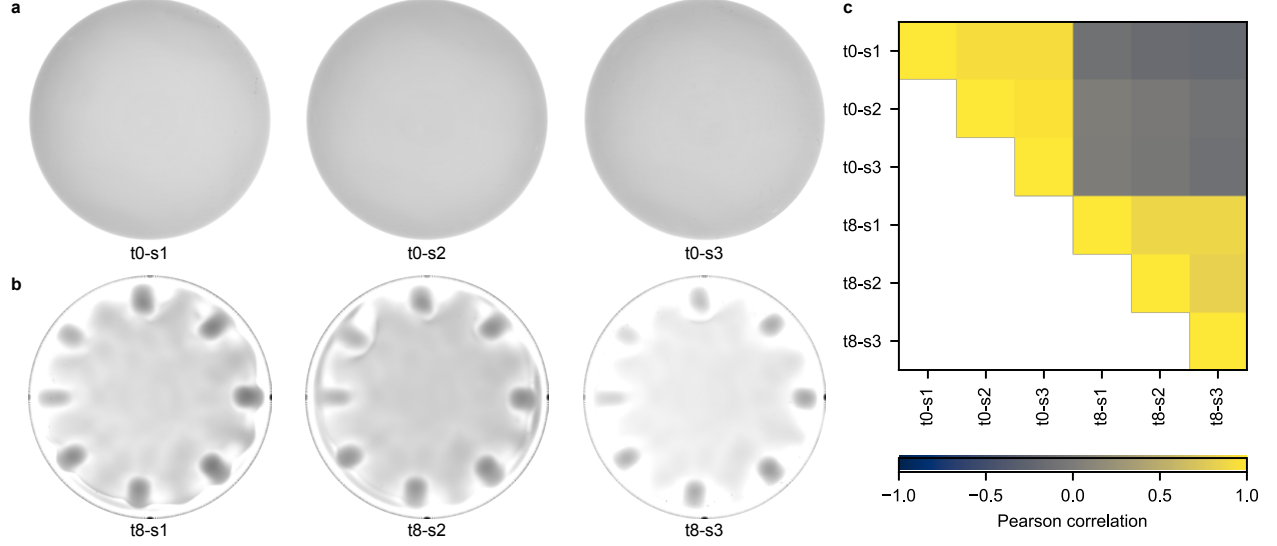
1.6 Mixed-mode imperfections

To demonstrate the ability of the fabrication method to generate mixed-mode and non-periodic imperfection fields, we fabricated additional shells under broadband and time-varying excitation conditions. Rather than applying single-frequency sinusoidal signals, the mold is driven using audio signals containing a wide range of frequencies and temporal modulations. This approach results in highly diverse, non-axisymmetric thickness patterns that differ qualitatively from the single-mode cases presented in the main text.

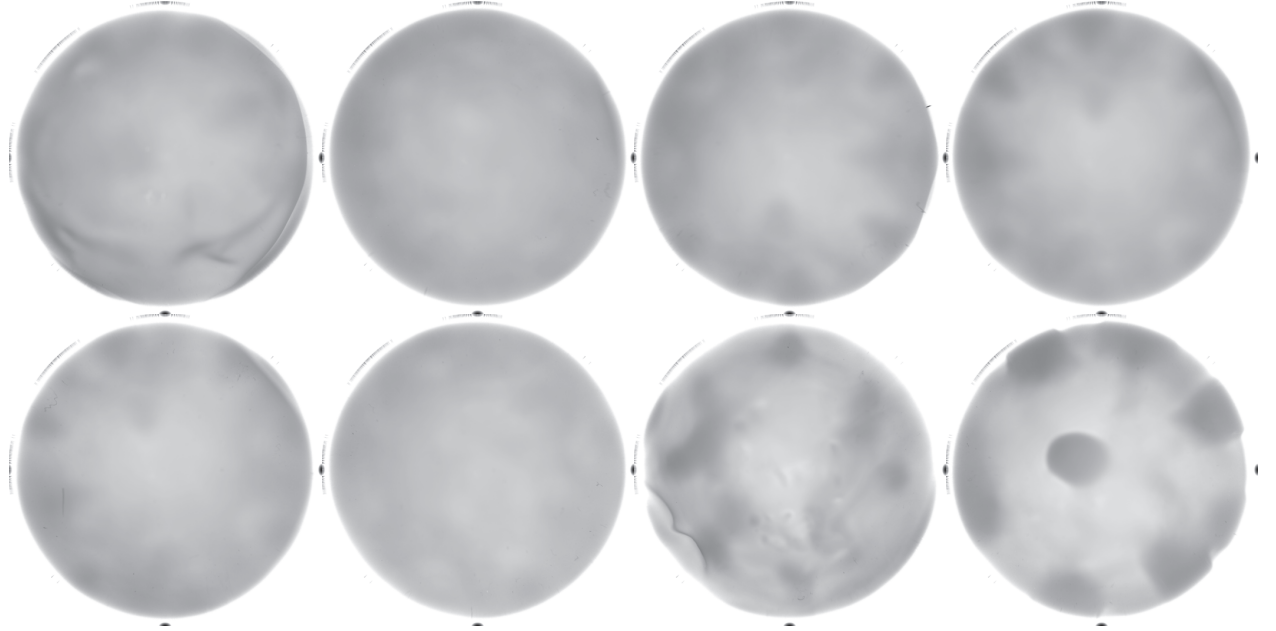
Representative examples include excitation using music spanning different genres and spectral contents, which serve here as convenient broadband inputs. The resulting imperfection fields exhibit both features resembling individual modal patterns and previously unobserved spatial organizations (S.Fig. 4).

1.7 Imperfection geometries through mold design

To demonstrate that the vibration-assisted fabrication method is not limited to standard modal shapes, we fabricate elastic molds with pre-designed geometric features. Rather than casting



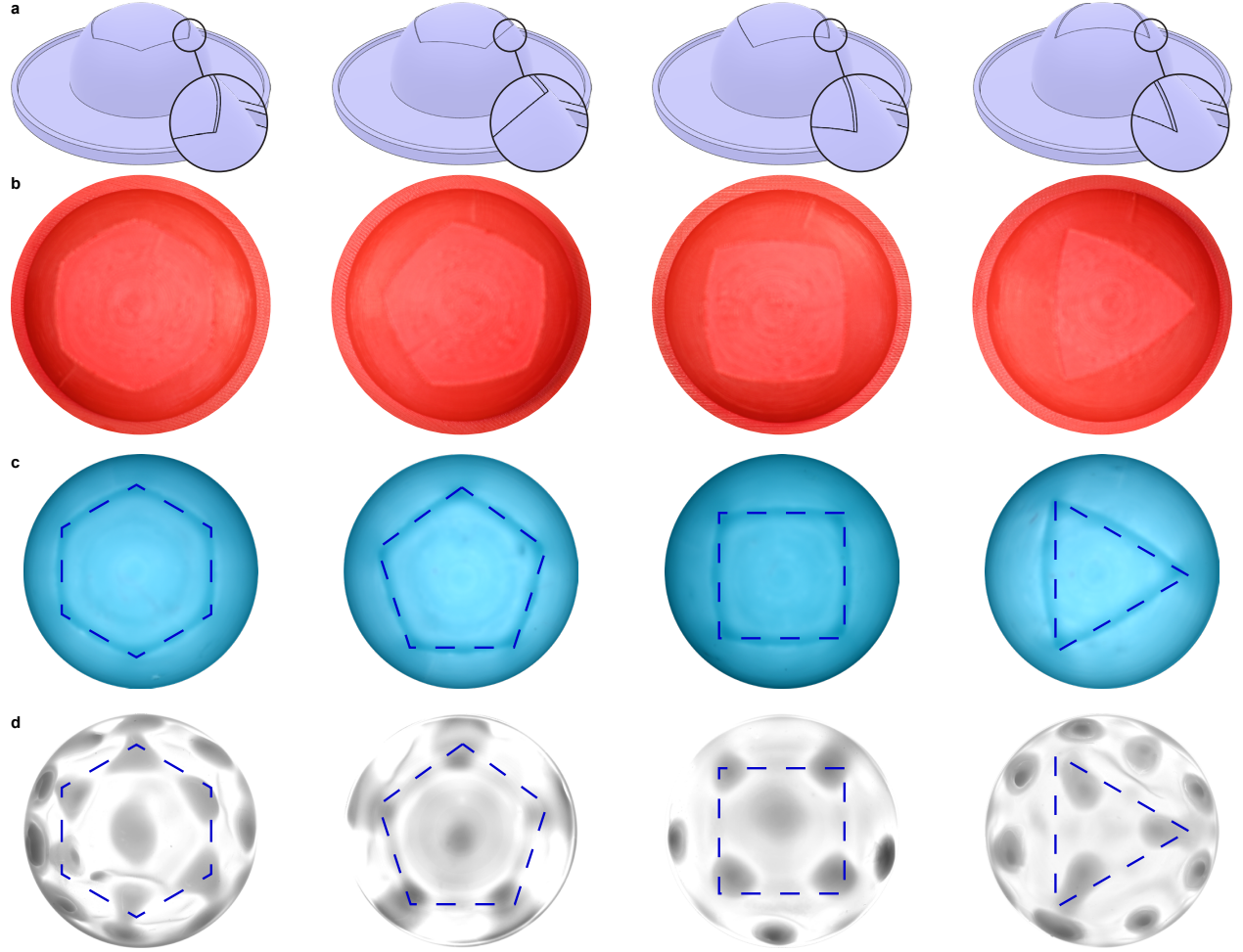
Supplementary Figure 3: **Shape comparison between shells based on photographic evidence.** a. Three near-perfect shells and b. three shells with 8 bump-like defects are compared. c. The resulting Pearson correlation is plotted showing close correlation between shells within each group, and minimal correlation across groups. Source data are provided as a Source Data file.



Supplementary Figure 4: **Mixed-mode imperfections.** Eight different shells fabricated using sequential broadband and time-varying excitation signals.

the elastic molds through repeated coating of a smooth metal sphere, we 3D print hemispherical negatives containing polygonal indentations (triangular, square, pentagonal and hexagonal, see S.Fig. 5a,b). These negatives are used to fabricate elastic molds with corresponding localized

thickness increase on their underside (S.Fig. 5c). The rest of the fabrication procedure remains identical: the mold is mounted onto the speaker, and silicone is poured as the mold is excited at a single frequency. The resulting cast shells (S.Fig. 5d) demonstrate that material accumulation concentrates at the antinodes of the modified molds, which coincide with the vertices of the polygonal features. This produces imperfection patterns that break the symmetry of standard vibrational modes, including configurations with odd numbers of bump-like defects that are not achievable with the hemispherical mold.

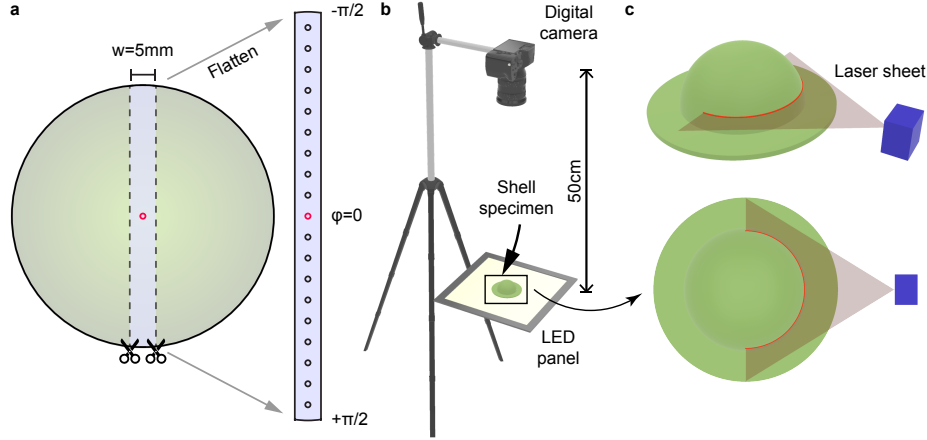


Supplementary Figure 5: **Casting using elastic molds with polygonal features.** a,b. Design and 3D printed hemispherical negatives with triangular, square, pentagonal and hexagonal indentations of 0.5 mm. c. Elastic molds where thickness is locally increased from the underside. d. The resulting cast shells showing that material accumulation is governed by the vertices of the polygonal features.

2 Measuring fabricated shell thickness profile

2.1 Destructive method

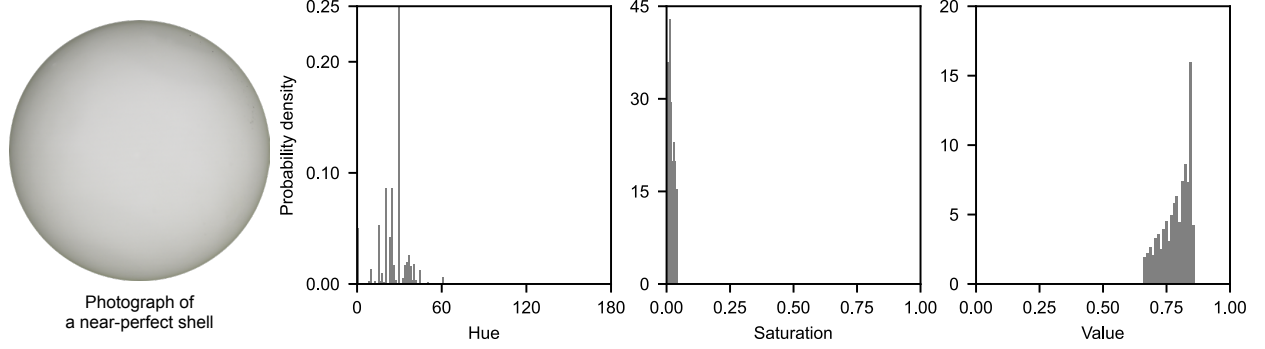
First we discuss the use of destructive measurement methods to capture the thickness profile of fabricated shells. To accurately measure the thickness variations in the shells, we extract strip segments (S.Fig. 6a) and measure the cross section thickness as a function of the polar angle using an optical microscope. We conduct this with three near-perfect shells fabricated without acoustic excitation (*i.e.*, static casts) but otherwise using the identical setup as our subsequent fabrication protocols (*i.e.*, thick elastic hemispherical mold atop the speaker). The same destructive measurement method is applied to specimens featuring 8 visible bump-like defects. These measurements serve as the ground-truth to correlate pixel intensity in photographs with thickness values.



Supplementary Figure 6: **Destructive and non-destructive means of shell thickness measurement.** a. The segment extracted from the near-perfect shells. The segment is flattened and its thickness is measured from end to end, covering polar angles from $\phi = [-\pi/2, \pi/2]$. b. Photography setup to capture shell thickness profile. c. Schematic setup of laser sheet positioning in relation to the hemispherical shells.

2.2 Photographic method

Next we inferred the shells' thickness profiles non-destructively through photography. Here, we ensure that the photography conditions including lighting, placement of specimens, and camera setup remain identical across all photos (S.Fig. 6b). With photography, each fabricated shell, both near-perfect and with-defects, is placed atop an LED panel (HSK A4 LED Light Box) in an otherwise dark room and photographed using a DSLR camera (Nikon D780) from the top at a fixed distance (0.5 meters) using a prime lens (Nikkor 105mm f/2.8G). From the resulting photos, we note that they are largely monochromatic, where there is limited Hue (H) and Saturation (S) distribution (see S.Fig. 7). This allows us to convert the Value (V) to greyscale, where each pixel has an intensity value I , without ambiguity. We then proceed to calibrate the images, and construct a correlation between the thickness profile of the shell and the greyscale value of the image ($0 \leq I \leq 1$ where 0 is black and 1 is white).



Supplementary Figure 7: **Analysis of a photographed shell showing the distribution of Hue, Saturation and Value.** The narrow distribution of Hue and Saturation allows the image to be converted to greyscale with minimal ambiguity. Source data are provided as a Source Data file.

2.3 Calibration photographs and inference of thickness

For calibration, we observe that the photographs of near-perfect shells show a progressive darkening towards the edge (see S.Fig. 8a). In physical measurements as well as in literature [1], this trend is much less pronounced for the silicone we used (Zhermack Elite Double Fast, Double 32). Rather, this optical effect arises from geometry rather than material variation. Under illumination, rays passing through the shell at the pole intersect the surface approximately orthogonally, whereas rays near the periphery traverse the shell at increasingly oblique angles. As a result, the optical path length through the material increases approximately as $1/\cos\phi$ where ϕ is the polar angle. Because transmitted intensity decays exponentially with optical path length according to Beer–Lambert attenuation, this geometric increase in path length leads to systematic edge darkening. We define a multiplicative correction filter F such that the corrected intensity $I_c(\phi)$ equals to $I_{\text{original}}(\phi) \cdot \xi(\phi)$, where

$$\xi(\phi) = \exp \left[K \left(\frac{1}{\cos\phi} - 1 \right) \right]. \quad (2)$$

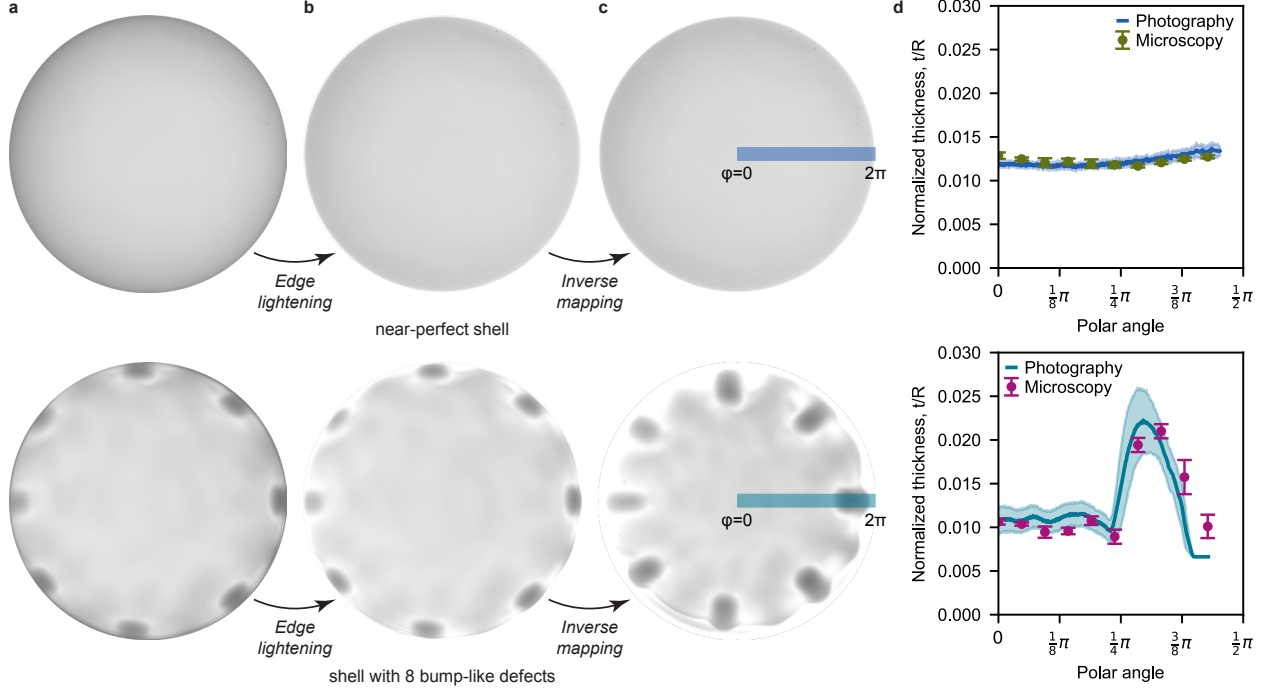
The correction strength factor K is chosen to empirically fit the photographed thickness of the near-perfect shells to the direct microscopy measurements and remains constant across all images (see S.Fig. 8b).

Next, the photographs are inverse-orthographically remapped to remove distortion arising from projecting a hemisphere (of the shell) to the flat plane of the camera sensor (Fig.8c). Under orthographical mapping, the pixels are expanded around the equator and/or compressed near the zenith in the radial direction as a function of the polar angle. To correct for this, the new radial coordinate of the inverse mapping equals to

$$r_{\text{new}} = 2 \frac{R}{\pi} \arcsin \frac{r}{R}, \quad (3)$$

where R is the radius of the hemisphere.

Finally, to correlate thickness, t , with respect to light intensity, we refer again to the Beer–Lambert law for a uniform material, $I \approx I_0 \exp -\mu t$, where with μ is the attenuation coefficient, I is the



Supplementary Figure 8: **Processing of photographed shells, both near-perfect and with defects.** a. Greyscale version of the photographs, b. corrected for edge darkening, c. inverse orthographic remapping and d. thickness profile along one azimuthal angle across polar angles from 0 to $\pi/2$ as directly measured and as inferred through photography. Source data are provided as a Source Data file.

intensity and I_0 is the incident intensity. Rearranging to isolate the thickness,

$$t \approx -\frac{1}{\mu} \log \frac{I}{I_0}, \quad (4)$$

and we fit the attenuation coefficient and the incident intensity I_0 to the measurements of the shells discussed above. Specifically, they are empirically fitted to the destructive measurements of the imperfect shells, in particular, the ones with 8 bump-like defects. This is due to the fact that imperfect shells possess a larger range of thickness values whose extrema are thinner and thicker than the near-perfect shells at different points on the shell. As a result, the attenuation coefficient $\mu = 1.40 \text{ mm}^{-1}$ and $I_0 = 1.264$. Using this, we plot the thickness profile along a single azimuthal angle picked to align with one of the bump-like defects, from zenith, $\phi = 0$, to the horizon, $\phi = \pi/2$, for near-perfect shells and for ones with 8 bump-like defects (S.Fig. 8d). The accuracy of the photography method allows us to rapidly measure the thickness of shells without further need of destructive methods or CT scanning.

3 Finite-element modal analysis

Finite element (FE) simulations are conducted using ABAQUS to analyze the dynamical behavior of hemispherical molds. Specifically, the simulations are used to compute natural frequencies and mode shapes.

A geometrically perfect hemisphere is defined in spherical coordinates with radius $R = 25.4$ mm, spanning the full hemisphere domain ($\theta \in [0, \pi/2], \phi \in [0, 2\pi]$). The shell thickness is set to $t = 0.7$ mm. The geometry is discretized using reduced-integration quadrilateral shell elements (S4R), with approximately 8100 elements used in total. The equatorial boundary is constrained in all degrees of freedom to emulate the experimental condition.

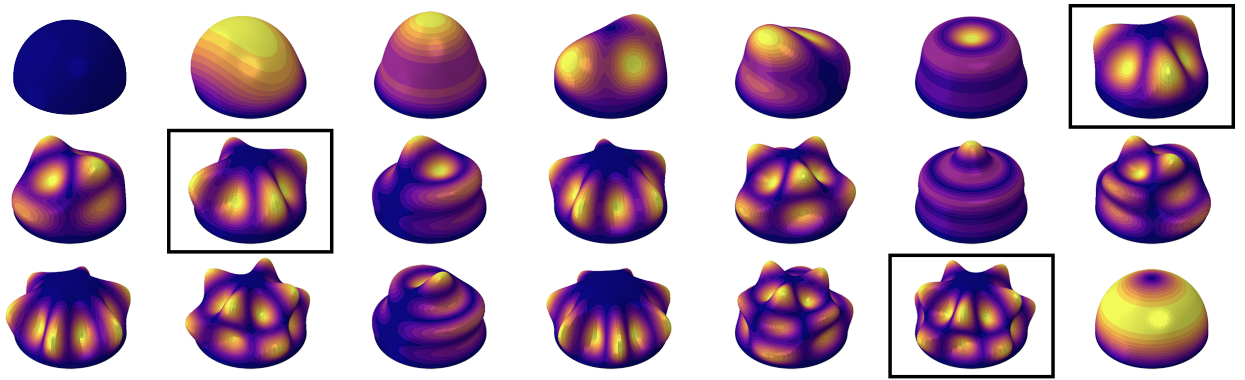
A linear perturbation step is used to extract the first 30 eigenmodes of the hemispherical shell. The *Lanczos* eigensolver is employed. The computed mode shapes characterize the shell’s natural vibration patterns and are used as a basis for interpreting deformation patterns observed during acoustic excitation (S.Fig. 9).

3.1 Inference of shell thickness from FE analysis

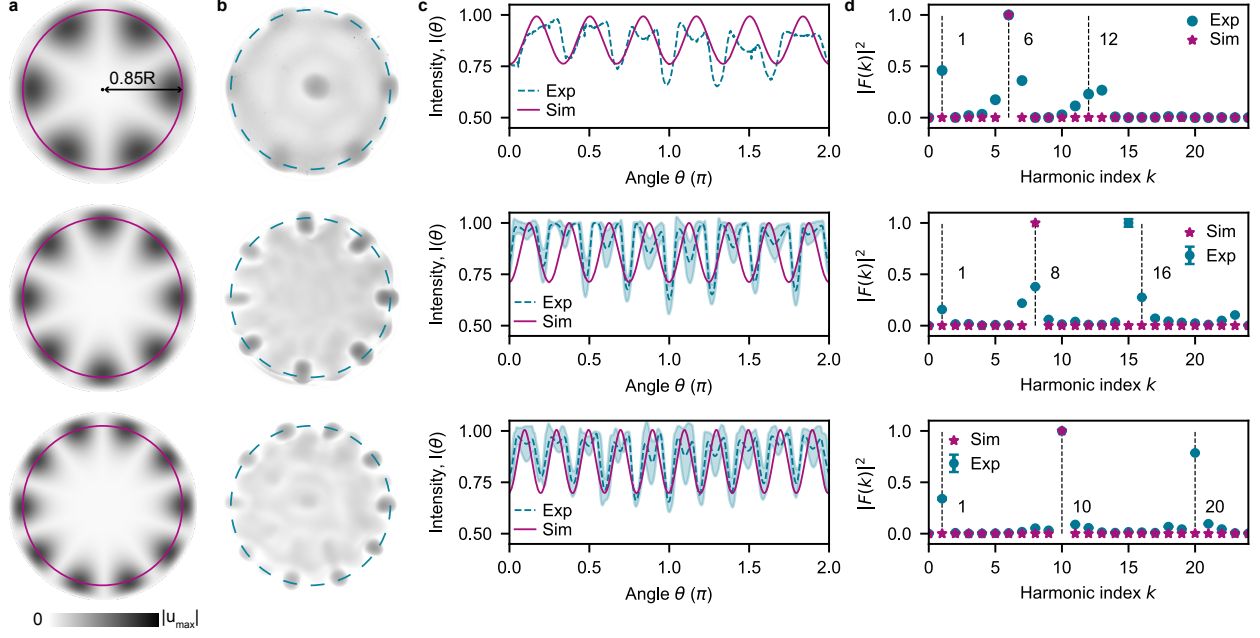
Here, we measure the similarity between the photographs of the fabricated shells and the resulting shapes of the mold’s normal mode. We elect to conduct linear frequency extraction analysis to demonstrate that the specimens can emulate the shape of vibrational modes of the elastic molds. Since all vibrations can be represented as a superposition of natural modes (exactly in the linear case, approximately otherwise), by showing correlation between the fabricated specimens and the FE modal shapes, we demonstrate the versatility of the fabrication method.

In Abaqus, the eigenvectors are normalized such that the maximum of each displacement component is 1. The magnitude of displacement is visualized using greyscale where white and black correspond to zero and maximum displacements respectively. As a result of the above analysis on the fluidic motion, this greyscale value is squared. The resulting images are scaled with the same number of pixels as the photographs of the fabricated shells. Note that we aim to correlate two different physical quantities, namely the modal displacements of the mold and the thickness of the shells fabricated using the mold. Therefore, rather than comparing the images on a pixel-level, we measure the similarities in the features of the images. Specifically, we assess whether the defects are present at the antinodes of the modal shape.

We proceed to extract three modal shapes at the requested frequencies; these feature 6, 8 and 10 bump-like defects respectively (S.Fig. 10a). The displacement fields resulting from FE modal analysis are plotted alongside the photographs of the imperfect shells (S.Fig. 10b). We plot the amplitudes of the intensity as a function of the azimuthal angle around a fixed radius ($0.85R$) to showcase the similarity between the two (S.Fig. 10c). The RMSE between experimental and



Supplementary Figure 9: **The first 21 unique modal shapes.** The three modes selected for fabrication are highlighted.



Supplementary Figure 10: **Comparison between modal shapes from FE modal analysis and photographs of imperfect shells.** a. Three modal shapes showing 6, 8, and 10 antinodes in a ring near the perimeter where the greyscale represents normalized displacement magnitudes. b. Photographs of fabricated shells with 6, 8 and 10 bump-like imperfections. c. Intensity plots around a ring at a radius of $0.85R$. d) Power spectra showing the magnitude of patterns with different periodicities. Source data are provided as a Source Data file.

simulation intensity measurements for the three specimens are 0.0798, 0.127, 0.0924 respectively. We further observe slight thickening at the nodes of the modal shape as well. To quantify periodicity, we analyze intensity variations using a Fourier transform. Each image is first reduced to a one-dimensional signal by averaging pixel intensities over a circular ring near the defects ($0.85 \pm 0.05R$), yielding an intensity profile I . The profile is then mean-subtracted to remove the zero-frequency component and transformed using discrete Fourier transform.

The resulting Fourier coefficients decompose the signal into harmonics corresponding to periodic variations around the circumference. The magnitude of the coefficient of the power spectrum $|F(k)|^2$ at harmonic index k quantifies the strength of a pattern repeating k times over one revolution. When this is applied to the FE modal displacement, one clear peak is present, representing the number of antinodes in that particular mode (S.Fig. 10d). For the fabricated specimens, the power spectrum similarly shows a local maximum at the requested number of defects. However, other local maxima are also present, specifically, at $k = 1$ and at higher harmonics, *e.g.*, $k = 12$ for the specimens with 6 bump-like defects. The harmonic index of 1 reflects a global imperfection where one side of the hemisphere is thicker than the other. This can be attributed to the imprecision when mounting the mold onto the speaker and the acoustic quality of the speaker itself. The higher harmonics arise from the non-sinusoidal intensity profiles. The indices around the peaks can be attributed to nonlinearities in the acoustic forcing and geometrically nonlinear structural response at high excitation amplitudes. Altogether, these show that the imprinted imperfections can faithfully retain the feature of the natural vibrational shapes from FE modal analysis.

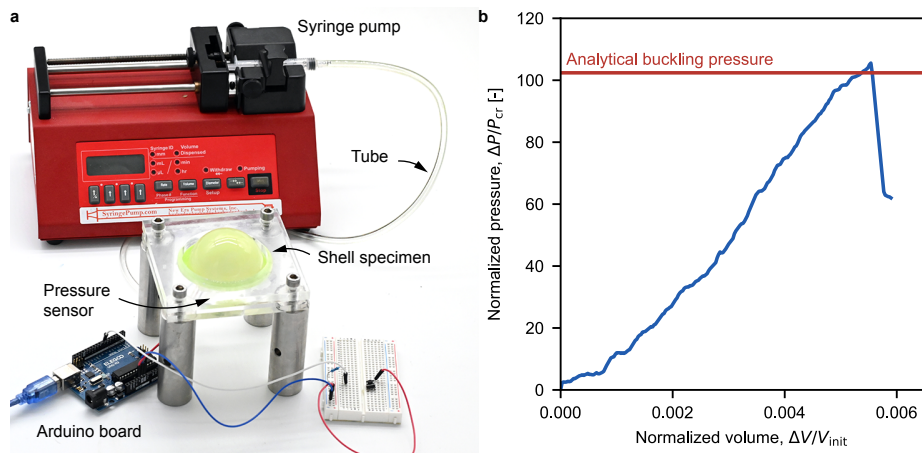
The above analyses show that the features of the thickness profile resulting from vibration-assisted casting process can be indirectly inferred from the FE modal shape. To do so, one would map the displacement field to greyscale intensity, then apply Eq. 4 to calculate the resulting thickness.

4 Mechanical characterization of hemispherical shells

4.1 Experimental setup

We characterize the pressure–volume relationship of the shell under partial vacuum loading. The shell is placed over an acrylic (PMMA) plate with a center hole connected to a syringe pump via a tube. To ensure an airtight seal, a second acrylic plate is positioned above the shell, and the two plates are clamped together with screws, forming a closed system consisting of the air inside the shell, the connecting tubing and the 1 mL syringe on the syringe pump.

Internal pressure is monitored using an MPL3115A2 barometric sensor. Data from the sensor is recorded in real-time with an ELEGOO UNO R3 microcontroller throughout the loading process at a period of 0.8 s.



Supplementary Figure 11: **Mechanical testing of thin elastomeric shells.** a. Automated setup to extract air from the interior of the hemispherical shells while simultaneously recording its internal pressure. b. The pressure volume behavior of a near-perfect shell. Source data are provided as a Source Data file.

As the volume inside the closed system expands, the internal volume increases, and the pressure gradually decreases. This causes a pressure difference between the enclosed system and the surrounding. When the pressure difference exceeds a critical threshold, the shell loses stability and buckles, as indicated by the abrupt change in the pressure–volume response.

4.2 Calculation of shell volume

Since the system comprising of the air within the shell, the connected tube and the syringe is closed, and the temperature remains constant throughout the experiment, the pressure–volume relationship can be modeled using the ideal gas law. Under these conditions, the evolution of pressure and volume follows the equation, $p_0 v_0 = p_c v_c$, where p_0 and v_0 are the pressure and the volume of the air in the closed system at the initial state, and p_c and v_c represent the pressure and

the volume at subsequent points in time. Thus, we have,

$$p_0/p_c = v_c/v_0. \quad (5)$$

The total volume is the summation of the volume of the air in the syringe, in the tube and under the shell, *i.e.*, $V^{\text{Total}} = V^{\text{Sy}} + V^{\text{Tu}} + V^{\text{Sh}}$.

We can write Eq. 5, as

$$p_0/p_c = \frac{V_c^{\text{Sy}} + V_c^{\text{Tu}} + V_c^{\text{Sh}}}{V_0^{\text{Sy}} + V_0^{\text{Tu}} + V_0^{\text{Sh}}}, \quad (6)$$

which, after rearrangement, gives,

$$V_c^{\text{Sh}} = \frac{p_0}{p_c}(V_0^{\text{Sy}} + V_0^{\text{Tu}} + V_0^{\text{Sh}}) - (V_c^{\text{Sy}} + V_c^{\text{Tu}}). \quad (7)$$

Since at the initial state there is no volume in the syringe Eq. 7 can be simplified as,

$$V_c^{\text{Sh}} = \frac{p_0}{p_c}(V_0^{\text{Tu}} + V_0^{\text{Sh}}) - (V_c^{\text{Sy}} + V_0^{\text{Tu}}), \quad (8)$$

where, p_0 is the initial, or the atmospheric pressure, in the system, p_c is the pressure measured by the pressure sensor, V^{Sy} is the known volume in the syringe, V^{Tu} is the constant volume in the tube, *i.e.*, $V_0^{\text{Tu}} = V_c^{\text{Tu}}$ and V_c^{Sh} is the volume of air under the hemispherical shell. This allows us to plot the pressure-volume curves in Figure 5 of the manuscript.

4.3 Calculation of critical pressure of a perfect shell

For a geometrically perfect spherical shell under uniform compression, the critical buckling pressure is given by [2] as follows,

$$P_{\text{cr}} = \frac{2E}{\sqrt{3(1-\nu^2)}} \left(\frac{t}{R} \right)^2. \quad (9)$$

For the geometry and material in our study, we substitute the following, $E = 1.2 \text{ MPa}$, $\nu = 0.5$, $t = 0.2 \text{ mm}$, $R = 25.4 \text{ mm}$ and arrive at $P_{\text{critical}} = 102.4 \text{ Pa}$.

To verify that our fabrication prototype can reproduce shells that are geometrically near perfect, we turn off the acoustic perturbation. We experimentally test the shell using the method above and plot the resulting pressure volume curve in comparison to the analytical value (S.Fig. 11b).

4.4 Stiffness measures

The stiffness is measured from the quasi-linear portion of the normalized pressure-volume curves prior to the onset of instability is given in (Table 1). These normalized stiffness quantities are calculated as $E_{\text{norm}} = p_{\text{norm}}/V_{\text{norm}}$ over the initial linear regime ($V_{\text{norm}} = 0.002$). They are plotted in Fig. 5b of the manuscript.

To quantify the reduction in structural capacity with increasing excitation amplitude, we fit the measured knockdown factors to an exponential decay model $\kappa = \alpha \exp(-k \cdot v)$, where v is the normalized speaker volume, α is a shared pre-factor common to all modal families, and k is a mode-dependent decay constant. The fitting is performed using nonlinear least-squares regression (`scipy.optimize.curve_fit`).

Table 1: Linearized stiffness measure of the different shell specimens.

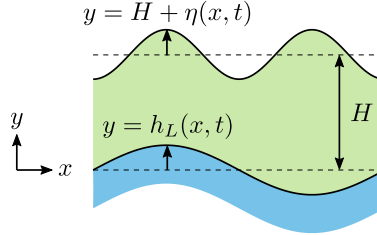
	No volume [–]	Volume 1 [–]	Volume 2 [–]	Volume 3 [–]
Near-perfect shell	119.7 ± 10.54			
6 bump-like defects		115.5 ± 14.09	115.1 ± 7.685	72.21 ± 12.61
8 bump-like defects		133.5 ± 9.091	112.1 ± 1.304	107.9 ± 8.599
10 bump-like defects		113.3 ± 8.810	109.6 ± 6.624	104.2 ± 3.334

5 Analysis of fluid motion

The vibration of the mold drives a flow in the liquid silicone layer that is ultimately responsible for the steady deflection of the liquid silicone’s free surface. To gain insight into the flow mechanisms that lead to the steady wavy free surface, we develop and analyze a minimal model that captures the essential physics of the problem. We particularly wish to understand why the free surface’s wavenumber is twice that of the mold’s vibration pattern, and why the free surface deflects outward at the antinodes of the mold’s vibration.

Consider a two-dimensional layer of liquid of nominal thickness H that is bounded below by a vibrating surface and above by a free surface (S.Fig. 12). The bottom boundary has a prescribed deformation, and the free surface deforms in response to the flow. We take the bottom boundary’s deflection to be given by a standing wave of frequency ω , wavenumber k , and amplitude ϵ ,

$$h_L(x, t) = \epsilon \sin(\omega t) \cos(kx). \quad (10)$$



Supplementary Figure 12: **Model problem for the flow of the liquid silicone.** A fluid (green) with a free surface is driven by the motion of a solid boundary (blue).

The liquid silicone in the experiment is non-Newtonian, but its viscosity is weakly affected by shear [1, 3], so we treat it as Newtonian for simplicity. Additionally, although the liquid silicone’s properties change as it cures, this occurs on a time scale that is much slower than any time scales relevant to the flow, so we treat the liquid’s properties as constant.

We perform a weakly nonlinear analysis to gain insight into the flow. This approach has been used in the past to gain insight into other flows driven by vibrating boundaries [4]. Lengths are scaled by H , time by ω^{-1} , velocity by ωH , and pressure by $\mu\omega$, with μ the fluid’s viscosity. From here onward, all quantities are dimensionless. We expand the fluid velocity and pressure fields, as well as the deflection of the free surface η from its mean height, in powers of $\epsilon^* = \epsilon/H$,

$$\mathbf{u} = \epsilon^* \mathbf{u}_1 + \epsilon^{*2} \mathbf{u}_2 + \mathcal{O}(\epsilon^{*3}), \quad (11)$$

$$p = \epsilon^* p_1 + \epsilon^{*2} p_2 + \mathcal{O}(\epsilon^{*3}), \quad (12)$$

$$\eta = \epsilon^* \eta_1 + \epsilon^{*2} \eta_2 + \mathcal{O}(\epsilon^{*3}). \quad (13)$$

Substituting these expansions into the Navier-Stokes equations and grouping terms of the same order, the primary flow satisfies

$$\nabla \cdot \mathbf{u}_1 = 0, \quad (14)$$

$$Re \partial_t \mathbf{u}_1 = -\nabla p_1 + \nabla^2 \mathbf{u}_1. \quad (15)$$

The secondary flow satisfies

$$\nabla \cdot \mathbf{u}_2 = 0, \quad (16)$$

$$Re \partial_t \mathbf{u}_2 = -\nabla p_2 + \nabla^2 \mathbf{u}_2 - Re \mathbf{u}_1 \cdot \nabla \mathbf{u}_1. \quad (17)$$

Note that the primary flow appears as a forcing term for the secondary flow. In the experiment, the Reynolds number $Re = \rho \omega H^2 / \mu$ is $\mathcal{O}(10^{-2})$, with ρ being the fluid's density.

The full flow field satisfies no-slip and no-penetration boundary conditions at the bottom boundary, while at the free surface it satisfies normal and tangential stress balances. Since in the experiment the free surface of the liquid silicone interfaces with air, the tangential stresses there can be ignored. Similarly, the normal stress balance simplifies to a balance between the normal stress in the liquid and the stress due to surface tension. Additionally, the free surface satisfies a kinematic boundary condition to ensure that its motion is consistent with that of the underlying fluid.

For our purposes, it will suffice to analyze the case where the free surface is fixed to be flat. The resulting pressure distribution at the free surface will then indicate how the free surface would deform once allowed to.

We start by analyzing the primary flow. Its governing equations are all linear and homogeneous, as are its boundary conditions, except for the inhomogeneous no-slip/no-penetration condition at the bottom boundary, which in dimensionless form reads

$$\mathbf{u}_1 = \cos(t) \cos(k^* x) \hat{j} \quad \text{at } y = 0. \quad (18)$$

This simply reflects that the primary flow is driven entirely by the velocity of the bottom boundary. Because of the linearity of the first-order equations, all primary variables will be sinusoidal in time and space with dimensionless frequency 1 and wavenumber k^* . As a result, there is no mean flow, and the free surface does not have a mean deflection at first order. To determine the mean deflection of the free surface, we must move on to the secondary flow.

The secondary flow's governing equations and boundary conditions have inhomogeneities stemming from the primary flow. In particular, the time-averaged flow satisfies

$$0 = -\nabla \langle p_2 \rangle + \nabla^2 \langle \mathbf{u}_2 \rangle - Re \langle \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 \rangle, \quad (19)$$

where $\langle \cdot \rangle$ denotes time-averaging. Since the primary flow is sinusoidal in time, the associated Reynolds stresses (which lead to the forcing term above) are non-zero, and they create a mean secondary flow, which we will call the streaming flow. In addition, the streaming flow has one inhomogeneous boundary condition,

$$\langle \mathbf{u}_2 \rangle = -\frac{1}{\epsilon^*} \langle \partial_y \mathbf{u}_1 h_L \rangle = -\partial_y \langle \mathbf{u}_1 \sin(t) \rangle \cos(k^* x) \quad \text{at } y = 0, \quad (20)$$

which we will refer to as the boundary forcing. Since the x -component of the primary velocity is zero at $y = 0$, continuity implies that only the x -component of the above boundary condition for the streaming flow is inhomogeneous.

The pressure associated with the streaming flow leads to steady deformation of the free surface. If we can determine the spatial structure of the streaming flow's pressure field, then we can determine the form of the free surface's mean deflection. Doing so requires knowledge of the Reynolds stresses and boundary forcing, so we return to the primary flow.

Rather than precisely solving for the flow variables, we will instead deduce the structure of the free surface's mean deflection using physical arguments and the basic mathematical structure of the problem. Letting ψ denote the stream function, it follows that the primary flow's stream function takes the form

$$\psi_1(x, y, t) = f(y) \sin(k^*x + \phi_x) \cos(t + \phi_t), \quad (21)$$

where f is a yet-unknown function, and ϕ_x and ϕ_t are yet-unknown phases. Then the two velocity components are

$$u_1(x, y, t) = \partial_y \psi_1(x, y, t) = f'(y) \sin(k^*x + \phi_x) \cos(t + \phi_t), \quad (22)$$

$$v_1(x, y, t) = -\partial_x \psi_1(x, y, t) = -k^* f(y) \cos(k^*x + \phi_x) \cos(t + \phi_t). \quad (23)$$

The forcing due to the Reynolds stresses is then

$$-\langle \mathbf{u}_1 \cdot \nabla \mathbf{u}_1 \rangle = \frac{1}{4} k^* (f f'' - f'^2) \sin[2(k^*x + \phi_x)] \hat{i} - \frac{1}{2} k^{*2} f f' \hat{j}, \quad (24)$$

and the inhomogeneous part of the streaming flow's boundary condition is

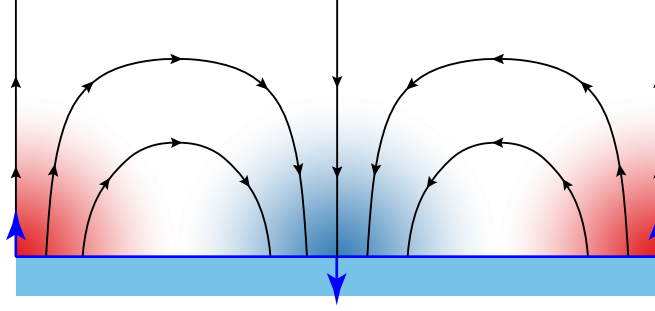
$$-\partial_y \langle u_1 \sin(t) \rangle \cos(k^*x)|_{y=0} = \frac{1}{2} f''(0) \sin(\phi_t) \left[\frac{1}{2} \sin(2k^*x) \cos(\phi_x) + \cos^2(k^*x) \sin(\phi_x) \right]. \quad (25)$$

Two features stand out. The first is that the y -component of the forcing from the Reynolds stresses is constant in x , so it will not tell us anything about the mean deflection of the free surface. The second is that the x -component of the forcing from the Reynolds stresses is purely sinusoidal in x with a wavenumber $2k^*$; the same is true for the boundary forcing. Therefore, the mean deflection of the free surface will also be sinusoidal in x with a wavenumber $2k^*$. This is exactly what we see in the experiment: the mean deflection of the free surface has twice the wavenumber of the mold's vibration pattern. The remaining element left to determine is the position of the free surface's mean deflection pattern relative to the bottom boundary.

To this end, we appeal to physical arguments to first determine the spatial phase ϕ_x . Because the problem has a reflection symmetry in the x -direction, from the inhomogeneous boundary condition for the streaming flow we deduce that $\phi_x = 0$ (otherwise it would have a non-zero spatial mean). We can also come to this conclusion by considering the physics of the primary flow. Where the bottom boundary moves up, the fluid is pushed up with it, and where it moves down, the fluid is pulled down with it. By symmetry, the streamlines above the antinodes of the bottom boundary must be vertical. Recalling that the coordinate system is chosen so that the shape of the bottom boundary is $\propto \cos(k^*x)$, it follows that $\phi_x = 0$. Thus, the pattern of the forcing due to the Reynolds stresses, as well as the pattern of the boundary forcing, are $\propto \sin(2k^*x)$.

Knowing that the two forcings are $\propto \sin(2k^*x)$ is not enough, however, since the sign of the pre-factor determines the spatial phase of the forcings. To determine the pattern of the forcing, we require additional knowledge of the primary flow. Where the bottom boundary moves up, the fluid is pushed up with it, which requires an increased pressure locally. Conversely, where the bottom boundary moves down, the fluid is pulled down with it, which requires a decreased pressure locally. The resulting pressure gradient leads to a circulating flow, as sketched in S.Fig. 13.

The qualitative picture of the primary flow is sufficient to determine the pattern of the forcing. The x -component of the forcing due to the Reynolds stresses is the mean of $-\mathbf{u}_1 \cdot \nabla u_1$, which is the



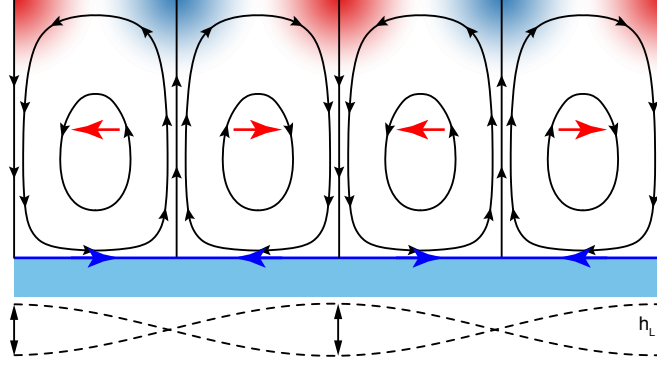
Supplementary Figure 13: **Sketch of the primary flow's pressure distribution along the vibrating lower boundary.** Red indicates high pressure while blue indicates low pressure. The blue arrows indicate the instantaneous motion of the bottom boundary. The pressure gradient leads to a circulating flow, as shown by the streamlines.

negative of the derivative of the horizontal velocity in the direction of the streamlines. Following the pattern of the streamlines, we deduce that the x -component of the forcing is directed toward the antinodes of the bottom boundary. Thus, the forcing due to the Reynolds stresses is $\propto \sin(2k^*x)$ with a negative pre-factor. The Reynolds stress forcing is shown with red arrows in S.Fig. 14. For the boundary forcing, we require $f''(0)$, which is related to $\partial_y u_1|_{y=0}$. From the pattern of the primary streamlines in S.Fig. 13, we deduce that $f''(0) > 0$. Additionally, we also require the temporal phase ϕ_t . For the low Reynolds number in the experiment, the low amount of inertia causes the primary flow to lag the motion of the boundary by a small amount, so ϕ_t is small and positive. (In fact, $\phi_t = \mathcal{O}(Re)$, so the boundary forcing and the forcing due to the Reynolds stresses are of the same order.) The resulting boundary condition for the streaming flow is shown with blue arrows in S.Fig. 14.

Because of the sinusoidal nature of the boundary forcing and the symmetry of the problem, the streamlines of the streaming flow must be vertical above the nodes and antinodes of the bottom boundary's vibration, with the direction of the flow as sketched in S.Fig. 14. From continuity, it follows that the flow takes on a circulatory nature, consisting of circulation cells of alternating sense. To determine the pressure distribution along the top surface, let us focus on the leftmost circulation cell in S.Fig. 14. As the fluid rises from the bottom, the forcing due to the Reynolds stresses accelerates the fluid leftward. Once the fluid starts to approach the upper-left corner of the cell, it must turn downward, which requires that it be accelerated rightward against the forcing of the Reynolds stresses. Thus, the pressure must increase locally above the antinode. The same logic applies to the other cells, leading to the pressure distribution sketched in S.Fig. 14: relatively high pressure above the antinodes, and relatively low pressure above the nodes.

To summarize, the vibration of the bottom boundary drives a time-periodic circulating flow in the fluid. This circulating flow has non-zero Reynolds stresses with twice the wavenumber that, together with a boundary forcing, create a streaming flow. The spatial patterns of the forcings and streaming flow lead to relatively high pressure above the bottom boundary's antinodes and relatively low pressure above the nodes. If the upper surface is allowed to move, it will respond to this pressure distribution, deflecting upward above the antinodes and downward above the nodes. In other words, fluid accumulates above the antinodes of the bottom boundary's vibration, just as in the experiment.

The mechanism described above sets the free surface into motion. Once it is in motion, it will modify the flow. Depending on the values of the parameters, the flow may take on a different form



Supplementary Figure 14: **Sketch of the streaming flow's pressure distribution along the free surface.** Red indicates high pressure while blue indicates low pressure. The blue arrows indicate the boundary condition at the lower boundary, which leads to the circulating flow shown by the streamlines. The red arrows show the x -component of the forcing due to the Reynolds stresses. The vibration pattern is shown at the bottom for reference.

than what has been sketched above. The governing dimensionless parameters are the Reynolds number Re , the capillary number $Ca = \mu\omega H/\gamma$ (where γ is the surface tension), the wavenumber k^* , and the amplitude of vibration ϵ^* of the bottom boundary. In the experiment, we have $Re = \mathcal{O}(10^{-2})$, $Ca = \mathcal{O}(10^2)$, and $k^* = \mathcal{O}(10^{-1})$. In this parameter regime, according to the weakly nonlinear theory, the dimensionless amplitude of deflection of the free surface scales as $A^* \sim Re^0 Ca^2 k^{*-2} \epsilon^{*2}$. In dimensional terms, the amplitude scales as $A \sim \lambda^2 \mu^2 \omega^2 \epsilon^2 \gamma^{-2} H^{-1}$, where we have used the wavelength λ of the vibration instead of the wavenumber k . Note that this assumes that the frequency, wavelength, and amplitude of the vibration can be controlled independently.

6 Supplementary Movies

Supplementary Movie 1 Casting procedure and visualization of the vibration and fluid accumulation

Supplementary Movie 2 Vibration of the elastic hemispherical mold as visualized using slow-motion videography

Supplementary Movie 3 Migration of the fluid as a result of the structural vibration as visualized using reflective particles

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